Statistics and Probability Letters 100 (2015) 93–97

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

L^1-Poincaré inequality for discrete time Markov chains[☆]

Lihua Zhang[a,*], Yingzhe Wang[b]

[a] School of Sciences, Beijing University of Posts and Telecommunications, Beijing, 100876, China
[b] School of Mathematical Sciences, Beijing Normal University, Beijing, 100875, China

ARTICLE INFO

Article history:
Received 25 March 2013
Received in revised form 16 January 2015
Accepted 18 January 2015
Available online 12 February 2015

MSC:
47J10
58C40
60J10

Keywords:
Discrete time Markov chain
Dobrushin coefficient
L^1-Poincaré inequality
L^2-Poincaré inequality

ABSTRACT

We introduce a new constant by L^1-Poincaré inequality which lies between the classical L^2-Poincaré constant and Dobrushin coefficient. Meanwhile, the bounds for the L^1-Poincaré constant are obtained by using Cheeger's technique.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

Consider a discrete time Markov chain $\{X_n : n \geq 0\}$ on a finite or countably infinite state space E. Let P be the transition probability matrix (or Markov Kernel) of X_n. And suppose that the Markov chain has a reversible probability μ. That is,

$$P(x, y) \geq 0, \quad \text{for all } x, y \in E, \quad \text{and} \quad \sum_{y \in E} P(x, y) = 1, \quad \text{for all } x \in E.$$

$$\mu(x)P(x, y) = \mu(y)P(y, x), \quad \text{for any } x, y \in E.$$

We assume throughout this article that P is irreducible.

As we know that ergodicity has been one of the research focuses of Markov processes. Using functional inequalities is one of effective ways to study ergodicity of Markov process (Deng and Song, 2012; Wuebker, 2012). It is well known that the classical L^2-Poincaré inequality is equivalent to exponential convergence of associated Markov semigroups, see references Bakry (2002), Chen (1996), Chen and Wang (2000) and Chen (2004). Although the relationship between the L^1-Poincaré inequality and the convergence rate of Markov chain is not clear, research about L^1-Poincaré inequality for Markov chain is still significant. Usually, the problem of inequality under the L^1 norm is often translated into a L^2 norm problem by using the Cauchy–Schwarz inequality (Diaconis, 2009; Saloff-Coste, 2004). Wang directly studied L^1-Poincaré inequality in Wang (2012) for continuous time Markov processes. However, the tools which are used in continuous time cases may not

☆ This work was supported by FRF for the Central Univ. (Grant No. BUPT2013RC0901) and 985 project.
* Corresponding author.
E-mail address: zhlh@bupt.edu.cn (L. Zhang).

http://dx.doi.org/10.1016/j.spl.2015.01.020
0167-7152/© 2015 Elsevier B.V. All rights reserved.

be useful in discrete time cases. Thus, exploring a new method to study L^1-Poincaré inequality for Markov chain in discrete time cases becomes important.

In this article, we introduce a new constant by L^1-Poincaré inequality which lies between the classical L^2-Poincaré constant and Dobrushin coefficient, which is always used to describe strong ergodicity of discrete time Markov chain (Anderson, 1991). And we obtain the estimation of optimal constant in L^1-Poincaré inequality by using Cheeger's technique and segmentation technique.

The definition of L^k-Poincaré inequality is following:

Definition 1.1. For $k = 1, 2$, we say that the L^k-Poincaré inequality of P holds if there exists a constant $0 < C_k < 1$ such that for all f with $\mu(f) = 0$, the following inequality holds

$$\mu(|Pf|^k) \le C_k \mu(|f|^k).$$

For $k = 1, 2$, we will denote by r_k the optimal constant in L^k-Poincaré inequality, i.e.

$$r_k := \sup\left\{\frac{\mu(|Pf|^k)}{\mu(|f|^k)} : \mu(f) = 0\right\}. \tag{1}$$

Theorem 1.1. *Assume that P is reversible, then*

$$r_2 \le r_1 \le \delta(P), \tag{2}$$

where $\delta(P)$ is the Dobrushin coefficient:

$$\delta(P) = \frac{1}{2}\sup_{x,y}\sum_z |P(x,z) - P(y,z)|.$$

We can use r_1 to describe the convergence of the semigroup P^n in $L^1(\mu)$.

Theorem 1.2. (a) $r_1 < 1 \Rightarrow \mu(|P^n f|) \le r_1^n \mu(|f|),\ \ \mu(f) = 0,$ *for all* $n > 1$.

(b) *Let*

$$r_1(P^m) := \sup\left\{\frac{\mu(|P^m f|)}{\mu(|f|)} : \mu(f) = 0\right\}, \quad m \ge 1.$$

Then $r_1(P^m) < 1 \Rightarrow \mu(|P^n f|) \le [r_1(P^m)]^{[\frac{n}{m}]}\mu(|f|),\ \ \mu(f) = 0,$ *for all* $n > m \ge 1$.

Finally, we define the other two constants related with the L^1-Poincaré inequality.

$$\gamma_0(A) := \sup\{\mu|Pf - \mu(f)| : f|_{A^c} = 0,\ \mu(|f|) = 1\}.$$

By using Cheeger's technique, we get upper bound and lower bound for r_1.

Theorem 1.3. *For the constants r_1 and $\gamma_0(A)$, we have*

$$\frac{1}{2}\sup_A \gamma_0(A) \le r_1 \le \inf_A\big(\gamma_0(A) \vee \gamma_0(A^c)\big). \tag{3}$$

Corollary 1.1. *Assume that P is reversible, then*

$$\frac{1}{2}\sup_x \sum_y |P(x,y) - \mu(y)| \le r_1 \le \inf_x\Big\{\sum_y |P(x,y) - \mu(y)| \vee \gamma(\{x\}^c)\Big\}.$$

2. Proof of the results

Proof of Theorem 1.1. In Theorem 1 in Zhang and Wang (2010), we proved that r_2 is an eigenvalue of P. Now, we assume g is a corresponding eigenfunction of r_2, i.e. $Pg = r_2 g$. Then, we have

$$\mu(g) = \mu(Pg) = \mu(r_2 g) = r_2\mu(g).$$

For $r_2 < 1$, it follows that $\mu(g) = 0$. According to the definition of r_1, we have

$$r_1 \ge \frac{\mu(|Pg|)}{\mu(|g|)} = \frac{r_2\mu(|g|)}{\mu(|g|)} = r_2.$$

So we prove the first part of inequality (2).

Download English Version:

https://daneshyari.com/en/article/1151408

Download Persian Version:

https://daneshyari.com/article/1151408

Daneshyari.com