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Semi-strong linearity testing in linear models with dependent but uncorrelated errors

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ABSTRACT

The covariance estimation of multivariate nonlinear processes is studied. The heteroscedasticity autocorrelation consistent (HAC) and White (1980) estimators are commonly used in the literature to take into account nonlinearities. Noting that the more general HAC estimation procedures may be sometimes viewed too sophisticated in applications, we propose tests for determining whether the simple White estimation could be used or if HAC estimation is necessary to ensure a correct statistical analysis of time series. The theoretical results are illustrated by mean of Monte Carlo experiments.

1. Introduction

Considerable attention has been paid to the analysis of stationary time series within the large framework of processes with dependent but uncorrelated innovations. Indeed such nonlinearities may arise for instance when the error process follows a GARCH (first introduced by Engle, 1982), all-pass (Andrews et al., 2006) or other models generally displaying a second order dependence (see Amendola and Francq, 2009). Note that such models are widely used in the literature. Examples where the errors are dependent can be found in Francq et al. (2005).

In many situations taking into account the nonlinearities leads to estimate asymptotic covariance matrices of the form

$$I_H = \sum_{h=-\infty}^{h=\infty} \mathrm{E}(\Upsilon_t \Upsilon'_{t-h}), \tag{1.1}$$

where the multivariate process (Υ_t) is specified below. Such matrices may be estimated using heteroscedastic autocorrelation consistent (HAC) methods (see the seminal paper of Newey and West (1987) for the kernel method, or Andrews and Monahan (1992) for the prewhitening method (VARHAC)). This may arise for instance when (Υ_t) is built from a process with dependent but uncorrelated innovations (the weak case). Reference can be made to Romano and Thombs (1996), Francq and Zakoïan (1998) in the univariate case, or Raïssi (2010), Boubacar Maïnassara (2012), Dufour and Pelletier (2008) in the multivariate case for the use of the HAC estimation. Nevertheless there are many cases where the expression of I_H may be simplified into $I_W := E(\Upsilon_t \Upsilon_t')$, so that the White (1980) method is preferable to estimate (1.1). This may arise (but not necessarily) when (Υ_t) is a martingale difference. For instance Chabot-Hallé and Duchesne (2008), Månsson and Shukur (2009) or Lee and Tse (1996), among others, proposed tools in presence of martingale differences innovations (the semi-strong case) which may lead to use I_W. Finally if we suppose that the innovations process is independent and identically

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distributed (i.i.d.) the statistical inference generally leads to standard asymptotic covariance forms. Note that considering a HAC covariance estimator induces sophisticated procedures, on the contrary to the White or standard methods. For instance the White estimator is simply computed by averaging the cross-products of the available Υ_t's, while HAC methods require to choose a bandwidth, or to fit a vector autoregressive model to the (possibly) high dimensional process (Υ_t). As a consequence a substantial gain in efficiency can be expected when using the White or standard covariance forms appropriately. Nevertheless when (1.1) cannot be simplified clearly the HAC methods must be used to obtain a convergent estimator.

In view of the above arguments it is important for the practitioners to have tools available for choosing the relevant covariance matrix estimation technique for the statistical analysis of time series. Francq and Zakoïan (2007) proposed a procedure for testing the equality of standard asymptotic covariances and asymptotic covariances of the form (1.1), in the framework of univariate weak autoregressive moving average (ARMA) models. The aim of the paper is to introduce a test for choosing between HAC or White covariance matrix estimation procedures. It is important to point out that we are not testing the semi-strong case versus the weak case.

In the paper the following notations are used. The almost sure convergence is denoted by $\xrightarrow{a.s.}$, while the convergence in distribution is denoted by $\Rightarrow$. The usual Kronecker product is denoted by $\otimes$. The vec(.) operator consists in stacking the columns of a matrix. For matrices A, B, C, D of appropriate dimensions and vectors z, w, basic rules give $(AB) \otimes (CD) = (A \otimes C)(B \otimes D)$ and $\mathrm{vec}(zw') = w \otimes z$.

2. The main result

Let us assume that (Υ_t) is given by $\Upsilon_t = \sum_{i=0}^{\infty} \Phi_i \epsilon_{t-i}$ where $\{\Phi_i\}$ is a sequence of real square matrices. The process (ϵ_t) is d-dimensional ($d \geq 1$). Define the strong mixing coefficients $\alpha_a(h)$ for a given stationary process (a_t)

$$\alpha_a(h) = \sup_{A \in \sigma(a_u, u \leq t), B \in \sigma(a_u, u \geq t+h)} |P(A \cap B) - P(A)P(B)|,$$

which measure the temporal dependence of the process (a_t). Let $\|a_t\|_q = (\mathrm{E}\|a_t\|^q)^{1/q}$, where $\|\cdot\|$ denotes the Euclidean norm with $\mathrm{E}\|a_t\|^q < \infty$ and $q \geq 1$. The following assumption delineates our framework:

Assumption A1. (i) The process (ϵ_t) is strictly stationary ergodic with finite positive definite covariance matrix Σ_ϵ, such that $\mathrm{E}(\epsilon_t) = 0$.
(ii) The process (ϵ_t) satisfies $\|\epsilon_t\|_{4+2\nu} < \infty$, and the mixing coefficients of the process (ϵ_t) are such that $\sum_{h=0}^{\infty} \{\alpha_\epsilon(h)\}^{\nu/(2+\nu)} < \infty$ for some $\nu > 0$.
(iii) Denoting by $\Phi_i^{k,j}$ the k,j-component of Φ_i, we assume that there exist constants $K > 0$ and $0 < \rho < 1$ such that $\sup_{k,j} |\Phi_i^{k,j}| < K\rho^i$ for all $1 \leq k, j \leq d$.

Assumption A1 allows for a wide range of specifications. In particular the process (ϵ_t) may be a martingale difference (as if it follows a GARCH model), uncorrelated but dependent (as in the case of All-Pass processes) or even correlated. The dependence structure is controlled by condition (ii).

In the literature the Υ_t's are often built from an (multivariate) observed process (X_t) following some model of the form:

$$X_t = m_{\theta_0}(X_{t-1}, X_{t-2}, \ldots, u_{t-1}, u_{t-2}, \ldots) + u_t, \tag{2.1}$$

where the conditional mean $m_{\theta_0}(.)$ is driven by a parameters vector $\theta_0 \in \mathbb{R}^\ell$. The stationary innovations process (u_t) is uncorrelated but possibly dependent. Typically the practitioners consider (multivariate) ARMA models for the conditional mean. The reader is referred to Francq et al. (2005) or Boubacar Maïnassara and Francq (2011) among others for the statistical analysis of ARMA models with uncorrelated but dependent errors which may lead to use HAC or White covariance estimators.

In order to exemplify, consider the testing of the linear Granger causality in mean widely used in time series econometrics to investigate links between variables. In most of applied works a VAR structure is assumed for this task:

$$X_t = A_{01}X_{t-1} + \cdots + A_{0p}X_{t-p} + u_t,$$

where the A_{0i}'s are the autoregressive matrices satisfying regularity conditions, so that we may write $X_t = \sum_{i=0}^{\infty} \phi_i u_{t-i}$, where ϕ_i is a sequence of matrices (see Lütkepohl (2005)). For building a Wald test for linear Granger causality in mean we consider $\Upsilon_t = \mathrm{vec}\left(u_t X_{t-1}^{p'}\right) = X_{t-1}^p \otimes u_t$, where $X_{t-1}^j := (X_{t-1}', \ldots, X_{t-j}')'$ for some $j > 0$. Writing $X_t^p = \widetilde{A}X_{t-1}^p + \tilde{u}_t = \sum_{i=0}^{\infty} \widetilde{A}^i \tilde{u}_{t-i}$ with

$$\widetilde{A} = \begin{pmatrix} A_{01} & \ldots & A_{0p} \\ I_d & 0 & 0 \\ & & \\ 0 & \ddots & 0 \\ 0 & 0 & I_d \end{pmatrix},$$

and $\tilde{u}_t = (u_t', 0, \ldots, 0)'$, we have $\Upsilon_t = \sum_{i=0}^{\infty} (\widetilde{A}^i \otimes I_d)(\tilde{u}_{t-i-1} \otimes u_t) = \sum_{i=0}^{\infty} \Phi_i \epsilon_{t-i}$ taking $\Phi_i = \widetilde{A}^i \otimes I_d$ and $\epsilon_{t-i} = \tilde{u}_{t-i-1} \otimes u_t$. Of course the autoregressive parameters are unknown and have to be estimated. As a consequence denoting by $\hat{u}_t$ the residuals of the estimation stage, we consider $\widehat{\Upsilon}_t = X_{t-1}^p \otimes \hat{u}_t$ in practice.