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A new strategy for optimal foldover two-level designs

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ABSTRACT

This paper studies the issue of optimal foldover plans for two-level designs in terms of the uniformity criteria measured by Lee, symmetric, wrap-around and centered discrepancy. An algorithm for searching the optimal foldover plans is also developed. New analytical expressions and new lower bounds for the above uniformity criteria of two-level combined designs are obtained.

1. Introduction

Fractional factorial designs are widely used by practitioners for screening experiments. However, one consequence of using fractional factorial designs is the alias structure of main effects or interactions. A common method to deal is to add more runs. A standard follow-up strategy discussed in many textbooks involves adding a second fraction, called a foldover design. Foldover of a fractional factorial design is a quick technique to create a design with twice as many runs, which typically releases aliased factors or interactions. A standard approach to foldover two-level fractional factorial designs is to reverse the plus and minus signs of one or more columns of the original design.

In the last few years, much attention has been paid in employing the discrepancy to assess the optimal foldover plans. The foldover plan such that the combined design (original design plus foldover) has the smallest discrepancy value over all foldover plans is called the optimal foldover plan (see Definition 1). The uniformity criterion has firstly been employed to assess the optimal foldover plan by Fang et al. (2003). Fang et al. (2003) firstly used the uniformity criterion measured by the centered L_2-discrepancy to search the optimal foldover plan. Lei et al. (2010) obtained some lower bounds of the centered L_2-discrepancy of combined designs when all Hamming distances between any distinct pair of runs of the two-level initial design are equal. Ou et al. (2011) obtained the lower bounds for centered L_2-discrepancy, symmetric L_2-discrepancy and wrap-around L_2-discrepancy of two-level combined designs for general case.

It is an important issue to find good lower bounds for the discrepancy measure of uniformity, because lower bounds can be used as benchmarks in searching for uniform (or optimal) designs. A design whose discrepancy value achieves a sharp

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lower bound is a uniform design with respect to this discrepancy. Recently, there has been considerable interests in trying to explore lower bounds for the discrepancy. Many authors tried to find good lower bounds for various discrepancies. Finally, Elsawah and Qin (2014, 2015b) obtained a new lower bound of the centered L_2-discrepancy under four-level U-type design and the first lower bound of the centered L_2-discrepancy for mixed two and three-level U-type design, respectively.

This paper studies the issue of optimal foldover plans for two-level designs in terms of the uniformity criteria measured by Lee discrepancy ($\mathcal{LD}$), symmetric L_2-discrepancy ($\mathcal{SD}$), wrap-around L_2-discrepancy ($\mathcal{WD}$) and centered L_2-discrepancy ($\mathcal{CD}$). For two-level fractional factorials as the original designs, we investigate new analytical expressions of the above discrepancies for symmetric two-level combined designs. This analysis provides new lower bounds of these discrepancies for symmetric two-level combined designs under general foldover plans, which can be used as benchmarks for searching optimal foldover plans. An algorithm for searching the optimal foldover plans is also developed. A comparison study between the new lower bounds and the previous lower bounds is provided by figures and numerical examples.

The remainder of this paper is organized as follows. Section 2 describes general structure of two-level combined designs under the above discrepancies. In Section 3, new formulations of the above discrepancies for two-level combined designs are obtained. In Section 4, new lower bounds of the above discrepancies for two-level combined designs are provided. An algorithm for searching the optimal foldover plans with examples is also developed in Section 5.

2. Uniformity criteria under foldover plans

While the method of reversing signs loses its efficacy when factors in the original design have more than two levels, our method is good for general s-level fractional factorial designs.

Consider a class of two-level U-type designs with n runs and m factors, denoted as $U(n; 2^m)$. A design d in $U(n; 2^m)$ can be presented as an $n \times m$ matrix with entries 0 and 1, with each element occurring equally often in each column. Let d be a design in $U(n; 2^m)$ and d is regarded as a set of m columns $d = (x^1, x^2, \ldots, x^m)$, where $x^j = (x_{1j}, \ldots, x_{nj})'$ is the jth column of d, $j = 1, \ldots, m$. Define $\Gamma = \{\gamma = (\gamma_1, \ldots, \gamma_m) | \gamma_j = 0, 1; 1 \le j \le m\}$, then for any $\gamma = (\gamma_1, \ldots, \gamma_m) \in \Gamma$, it defines a foldover plan for design $d \in U(n; 2^m)$, for $1 \le j \le m$, the jth factor is mapped to $(x_{1j} + \gamma_j, \ldots, x_{nj} + \gamma_j)'$ (mod 2).

For any design $d \in U(n; 2^m)$ and a foldover plan $\gamma \in \Gamma$, the foldover design, denoted by d_γ, is obtained by mapping the columns in d defined above according to the foldover plan γ. Thus, it is to be noted that each foldover design is generated by a foldover plan. The full design obtained by augmenting the runs of the foldover design d_γ to those of the original design d is called as combined design, denoted by $d(\gamma)$, that is, $d(\gamma) = (d'\ d'_\gamma)'$.

In this paper, we consider the $\mathcal{LD}$, $\mathcal{SD}$, $\mathcal{WD}$ and $\mathcal{CD}$ to evaluate the optimal foldover plan. For two-level design $d \in U(n; 2^m)$, its $\mathcal{LD}$, $\mathcal{SD}$, $\mathcal{WD}$ and $\mathcal{CD}$ values, denoted by $\mathcal{LD}(d)$, $\mathcal{SD}(d)$, $\mathcal{WD}(d)$ and $\mathcal{CD}(d)$, can be expressed in the following forms respectively

$$[\mathcal{LD}(d)]^2 = -\left(\frac{3}{4}\right)^m + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}\left[1-\alpha_{ij}^k\right],$$

$$[\mathcal{SD}(d)]^2 = \left(\frac{4}{3}\right)^m - 2\left(\frac{11}{8}\right)^m + \frac{2^m}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}(1-|u_{ik}-u_{jk}|),$$

$$[\mathcal{WD}(d)]^2 = -\left(\frac{4}{3}\right)^m + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}\left[\frac{3}{2} - |u_{ik}-u_{jk}|(1-|u_{ik}-u_{jk}|)\right]$$

and

$$[\mathcal{CD}(d)]^2 = \left(\frac{13}{12}\right)^m - \frac{2}{n}\sum_{i=1}^{n}\prod_{k=1}^{m}\left[1+\frac{1}{2}\left|u_{ik}-\frac{1}{2}\right| - \frac{1}{2}\left|u_{ik}-\frac{1}{2}\right|^2\right] + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}\left[1+\frac{1}{2}\left|u_{ik}-\frac{1}{2}\right| + \frac{1}{2}\left|u_{jk}-\frac{1}{2}\right| - \frac{1}{2}\left|u_{ik}-u_{jk}\right|\right],$$

where

$$\alpha_{ij}^k = \min\left\{\frac{|x_{ik}-x_{jk}|}{2}, 1-\frac{|x_{ik}-x_{jk}|}{2}\right\}, \qquad u_{jk} = \frac{2x_{jk}+1}{4} \quad \text{and} \quad x_{jk} \in \{0, 1\}.$$

The reader can refer to Zhou et al. (2008) and Hickernell (1998a,b).

Under a foldover plan γ, these discrepancies on the combined design $d(\gamma)$, denoted by $[\mathcal{LD}(d(\gamma))]^2$, $[\mathcal{SD}(d(\gamma))]^2$, $[\mathcal{WD}(d(\gamma))]^2$ and $[\mathcal{CD}(d(\gamma))]^2$, can be calculated by the following forms respectively

$$[\mathcal{LD}(d(\gamma))]^2 = -\left(\frac{3}{4}\right)^m + \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}\left[1-\alpha_{ij}^k\right] + \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\prod_{k=1}^{m}\left[1-\alpha_{ij}^k(\gamma_k)\right], \tag{2.1}$$