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Simulation of Gamma records

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ABSTRACT

In the present paper, we discuss algorithms of record generation when records are taken from a Gamma population. We develop such generation algorithms and study their asymptotic performance.

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1. Introduction

Let $X_1, X_2, \ldots$ be a sequence of random variables defined on a common probability space. Let us introduce the sequences of record times $L(n)$ and record values $X(n)$ as follows:

$$L(1) = 1,$$
$$L(n+1) = \min\left\{j : j > L(n),\, X_j > X_{L(n)}\right\}, \qquad X(n) = X_{L(n)} \quad \text{for } n \geq 1.$$

The theory of records is much discussed and well developed. One can refer here to the books of Arnold et al. (1998) and Nevzorov (2001). A new and interesting turn in this theory is associated with simulation of record data; see the works of Bairamov and Stepanov (2011), Luckett (2013), Nevzorov and Stepanov (2014) and Balakrishnan et al. (in press). See also simulation of concomitants of records in Stepanov et al. (2016).

The simulation of records is important for modeling such experiments where only record data is available. The concept of record sample was introduced in Samaniego and Whitaker (1986). One situation where such samples arise is in industrial stress and life-testing wherein measurements are made sequentially and only values larger than all previous values are recorded. For some other examples, see Hofmann and Nagaraja (2001), Hofmann (2004) and Balakrishnan and Stepanov (2006). It should also be noted that record samples are used in data reduction procedures.

The Gamma distribution is important for statistics and simulation. It belongs to the exponential family, and many research methods applicable to the Gamma distribution are also applicable to the other distributions of this family. Generating Gamma random variables, in its turn, is an interesting simulation problem. Several generation techniques are known presently; see, for example, Kundu and Gupta (2007).

In our paper, we discuss generation algorithms of Gamma records. The corresponding algorithms are based on the rejection method. Before presenting these algorithms we introduce the direct method of record generation by which records from any population can be generated.

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The direct method *The value $X(1) = X_1$ is generated and kept. For $n \geq 1$, one can apply the recursive approach, which assumes that $X(n)$ is already obtained. One then generates observations X_i till one of them, say X_j, is greater than $X(n)$. Then $X(n+1) = X_j$ becomes the next record value, which is also kept.*

It should be noted that this method is burdensome and slow, especially when a large number of records is needed.

Let in the following $X_1, X_2, \ldots$ be independent and identically distributed random variables with continuous distribution $F(x) = P(X \leq x)$. It is known that the sequence $X(1), X(2), \ldots$ forms a Markov chain and

$$P(X(n+1) \leq x_{n+1} \mid X(n) = x_n) = \frac{F(x_{n+1}) - F(x_n)}{1 - F(x_n)} \quad (x_{n+1} > x_n). \tag{1.1}$$

In the case when the inverse function F^{-1} can be obtained explicitly, one can apply the inverse-transform method for generating record values, wherein the corresponding simulation algorithms are based on identity (1.1). For instance, if F is the standard exponential distribution, then $X(n)$ is generated as $-\log(U_1 U_2 \ldots U_n)$, where U_i $(i = 1, \ldots, n)$ are generations of random numbers, i.e. the random variables U_i are independent and uniformly distributed on $(0, 1)$. For references on simulation of exponential records, see, for example, Bairamov and Stepanov (2011), Luckett (2013) and Balakrishnan et al. (in press).

In the case when the inverse F^{-1} cannot be found analytically, one should apply the rejection method. Obviously, the rejection method should be used for generating normal and Gamma records. An interesting study has been recently done in Balakrishnan et al. (in press) on the normal case. Stimulated by their research, we propose new techniques for generating records taken from a Gamma population.

The rest of our paper is as follows. Further in Section 2, we propose algorithms for generating records taken from a Gamma population. Although these algorithms are based on the rejection method, they work with time as algorithms based on the inverse-transform method. The corresponding generation methods are then speedy and effective. This technique allows to generate "long" sequences of Gamma records; see Section 3. In Section 3, we also compute by numerical integration the vector of means and the variance–covariance matrix of Gamma records and present visual comparison of these computations with their estimates based on record data. We additionally apply in Section 3 a goodness-of-fit test to check our simulation results. Finally, we formulate conclusions in Section 4.

2. Simulation algorithms

In our study, the algorithms of generation of record values will be based on the rejection method, which we briefly introduce here.

The rejection method Suppose that one can generate a random variable Y having density function g by the inverse-transform method. Suppose that X with density function f cannot be generated by the inverse-transform method and X and Y have the same support. One should find a constant $c > 1$ such that $c = \sup_x \frac{f(x)}{g(x)}$.

The algorithm

STEP 1: Generate $Y = y$ (with density function g) and a random number $U = u$.

STEP 2: If $u \leq \frac{f(x)}{cg(x)}$, set $X = y$. Otherwise, return to STEP 1.

It should be noted that the choice of a random variable Y is determined by the fact that $c > 1$ should get the smallest possible value. It is also known that the number of iterations in this method for obtaining X is a geometric random variable with mean c.

The rejection method is one of the basic methods for generating Gamma random variables. One can find the standard generation algorithms in the book of Ross (2006, pp. 73–75). For a more sophisticated generation technique, see the work of Kundu and Gupta (2007). For generation purposes without loss of generality the scale parameter of Gamma distributions can be taken to be one. We thank an anonymous referee for bringing this important point to our attention.

Let here and in the following $F(x \mid \alpha)$ be a Gamma distribution with shape parameter $\alpha > 0$ and $f(x \mid \alpha)$ be the corresponding density function, i.e.

$$f(x \mid \alpha) = \frac{x^{\alpha-1} e^{-x}}{\Gamma(\alpha)} \quad (x, \alpha > 0).$$

It follows from (1.1) that the conditional density of $X(n+1)$ given $X(n)$ has the following form

$$f_{X(n+1)|X(n)}(x_{n+1} \mid x_n, \alpha) = \frac{f(x_{n+1} \mid \alpha)}{1 - F(x_n \mid \alpha)} \quad (x_{n+1} > x_n), \tag{2.1}$$

where $F(x_n \mid \alpha) = \int_0^{x_n} f(u \mid \alpha) du$. We do not further discuss how to generate Gamma records when $\alpha = 1$, because a Gamma distribution with $\alpha = 1$ is the standard exponential distribution. The record generation algorithm in this case is well-known; see, for example, Bairamov and Stepanov (2011), Luckett (2013) and Balakrishnan et al. (in press). In Sections 2.1–2.2, we discuss algorithms of record generation assuming, respectively, that $0 < \alpha < 1$ and $\alpha > 1$.

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