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Bootstrapping the empirical distribution of a linear process

Farid El Ktaibi [a,*], B. Gail Ivanoff [a], Neville C. Weber [b]

[a] Department of Mathematics & Statistics, University of Ottawa, 585 King Edward, Ottawa ON K1N 6N5, Canada

[b] School of Mathematics & Statistics, University of Sydney, NSW 2006, Australia

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ABSTRACT

The validity of the moving block bootstrap for the empirical distribution of a short memory causal linear process is established under simple conditions that do not involve mixing or association. Sufficient conditions can be expressed in terms of the existence of moments of the innovations and summability of the coefficients of the linear model. Applications to one and two sample tests are discussed.

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1. Introduction

There is a vast literature on the asymptotic behaviour of the empirical process generated by a stationary sequence $(X_i, i \in \mathbf{Z})$ of random variables and the validity of associated bootstrap techniques. Almost always, conditions on mixing or association of the sequence are imposed. As noted in recent papers by Sharipov and Wendler (2012) and Radulović (2012), mixing conditions can be difficult to verify and can exclude linear processes. In this paper we establish the validity of the moving block bootstrap (MBB) for the empirical process associated with short memory causal linear processes.

In the case of causal linear processes

$$X_i = \sum_{j\geq 0} a_j \xi_{i-j}, \quad i \in \mathbf{Z}, \tag{1}$$

where $(\xi_j : j \in \mathbf{Z})$ is a sequence of independent and identically distributed (i.i.d.) random variables and $(a_j : j \in \mathbf{N})$ is a sequence of constants. If the innovations have finite variance, the process is said to have short memory if $\sum_{j=0}^{\infty} |a_j| < \infty$. This model includes a wide range of stationary ARMA time series models, many of which are not mixing. A classic example due to Ibragimov is the following: let the ξ_i's be i.i.d. $N(0, 1)$ and let a_i be the coefficient of z^i in the power series expansion of the function $h(z) = (1 - z)^p$, where $p > 4$ is non-integer. In this case, $|a_i| = O(i^{1-p})$ but (X_i) is not strong mixing (cf. Gorodetskii, 1977). Therefore, a different approach is needed to establish empirical and bootstrap empirical central limit theorems for linear processes.

* Corresponding author.
E-mail addresses: felkt081@uottawa.ca (F. El Ktaibi), givanoff@uottawa.ca (B. Gail Ivanoff), neville.weber@sydney.edu.au (N.C. Weber).

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Asymptotics for the empirical distribution of the linear process have been developed by Doukhan and Surgailis (1998) and by Ho and Hsing (1997) under different conditions involving the moments of the innovations (ξ_j) and the behaviour of the coefficients (a_j); mixing assumptions are not required. For the short memory case, Ho and Hsing (1997) assume finite fourth moments for the innovations. In Doukhan and Surgailis (1998), sufficient conditions for an invariance principle for the empirical process are expressed in terms of the existence of moments of *any* order for ξ_j's combined with a corresponding condition on summability of the coefficients (a_j).

Under suitable conditions the empirical distribution of the causal linear process converges to a Gaussian process. However the limiting covariance structure cannot be evaluated from a single realization of the process, which necessitates a bootstrap procedure for applications.

An excellent general reference for various bootstrap techniques for stationary sequences is Lahiri (2003). The moving block bootstrap (MBB) was originally introduced by Künsch (1989) and Liu and Singh (1992). A comprehensive review of the moving block bootstrap (MBB) is given by Radulović (2002); sufficient conditions for a bootstrap empirical CLT are given in terms of α- or β-mixing coefficients. More recently the disjoint block bootstrap (DBB) was revisited in Radulović (2009), proving that under mild β-mixing conditions, if an empirical CLT exists for a fixed class of functions satisfying a moment condition, then there is also a DBB empirical CLT. Other bootstrap techniques are discussed by Politis et al. (1999) and Bühlmann (2002). In particular, when the linear process is invertible and admits an autoregressive representation of order ∞, Bühlmann describes a sieve bootstrap procedure that uses a semiparametric AR(p) approximation of the linear process.

In the references above, and in most of the available literature, assumptions of mixing or association are made, or in the case of a sieve bootstrap, invertibility of the linear process is assumed. Two exceptions are Proposition 3.1 of Peligrad (1998) and Theorem 3 of Radulović (2012), where sufficient conditions for the MBB CLT for the mean are given without mixing assumptions. The key is convergence (a.s. or in probability) of the conditional variance of the normalized sum of the bootstrap sample. This is extended to an empirical MBB CLT in Theorem 2.2 of Peligrad (1998) where sufficient conditions for convergence of finite dimensional distributions and tightness are expressed in terms of moments of the empirical distribution of the stationary sequence. However, the applications given in Peligrad (1998) are restricted to strongly mixing or associated stationary sequences.

In what follows, we will see that conditions on the innovations and coefficients of the linear process similar to those of Doukhan and Surgailis (1998) are sufficient to ensure that Peligrad's moment conditions hold. These simple assumptions allow us to handle non-mixing linear processes. Furthermore, even when the linear process can be shown to be mixing, our conditions improve on the sharpest available sufficient mixing rates for the empirical MBB CLT. This will be discussed in more detail at the end of Section 2.

We proceed as follows. The main results appear in Section 2. After reviewing the empirical CLT of Doukhan and Surgailis (1998) we present the empirical CLT for the moving block bootstrap in Theorem 2.5. In Section 3 we indicate how the result can be applied to one and two-sample tests. The proof of Theorem 2.5 appears in Section 4.

2. Main results

2.1. The empirical CLT

As in Doukhan and Surgailis (1998), we will be assuming that infinitely many of a_j's are non-zero; otherwise, the process is m-dependent for some $m < \infty$, and therefore strongly mixing — a case where bootstrap techniques are well understood. Let F and F_ξ denote the respective distribution functions of X_0 and ξ_0. The empirical CLT is proven under the following assumptions.

Assumptions 2.1. 1. Let $\{a_j, j \in \mathbf{Z}\}$ be a sequence of non-random weights, infinitely many of which are non-zero, satisfying

$$\sum_{j\geq 0} |a_j|^\gamma < \infty \quad \text{for some } \gamma \in (0, 1].$$

2. There exist constants $C < \infty$ and $\Delta \in \left(\frac{2}{3}, 1\right]$ such that for all $u \in \mathbf{R}$

$$|E\exp(iu\xi_0)| \leq \frac{C}{(1+|u|)^\Delta}.$$

3. $E[|\xi_0|^{4\gamma}] < \infty$.

Note that X_i might not have finite expectation. Also, $Var(X_i) < \infty$ if and only if $E[\xi_0^2] < \infty$. The more general the moment condition in Assumption 2.1.3, the more restrictive the summability condition Assumption 2.1.1.

Assumption 2.1.2 (first introduced in Giraitis and Surgailis, 1994) implies that the distribution function of ξ_0 satisfies the Hölder condition $|F_\xi(x) - F_\xi(y)| < C|x-y|^\Delta$ and that there exists $h_0 < \infty$ such that if $h \geq h_0$, the distribution function of the partial sum $\sum_{j=0}^{h} a_j\xi_{i-j}$ is differentiable with a bounded density satisfying a uniform Lipschitz condition (cf Doukhan and Surgailis, 1998). We remark that Doukhan and Surgailis assumed that $\Delta \in \left(\frac{1}{2}, 1\right]$, but in fact there is a small error in the

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