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On the higher order product density functions of a Neyman–Scott cluster point process

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ABSTRACT

A general formula and upper bounds for the intensity-reweighted product density functions of Neyman–Scott processes are obtained. Analytical expressions are presented for the intensity-reweighted product and cumulant densities of the Thomas process, an important special case of Neyman–Scott processes.

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1. Introduction

The first and second-order product densities, namely the intensity and pair correlation functions, are widely used in statistical inference for Neyman–Scott processes (Møller and Waagepetersen, 2003; Tanaka et al., 2008; Waagepetersen, 2007; Waagepetersen and Guan, 2009). Due to their complicated kernel convolution type (see e.g. (7)), the higher order product density functions of Neyman–Scott processes have seldom been exclusively studied or used in data analysis, although some attempts have been made (Schladitz and Baddeley, 2000). Nevertheless, they mainly emerge in the asymptotic theory regarding estimation of the first or second-order product density functions or their parameters (Stoyan et al., 1993; Heinrich and Klein, 2011). In such cases, the up to fourth-order product density functions are required to meet certain assumptions (see e.g. Waagepetersen and Guan, 2009).

This paper provides a general formula (Lemma 1) and upper bounds for the higher order intensity-reweighted product density functions of Neyman–Scott processes. Particularly, closed-form analytical expressions (Theorem 1) are derived for the intensity-reweighted product density functions, as well as the cumulant density functions, of a Thomas process.

2. Neyman–Scott processes

Let Φ be a stationary Poisson process on $\mathbb{R}^d$, $d \geq 2$, with intensity $\kappa > 0$, $k : \mathbb{R}^d \to [0, \infty)$ be a kernel (density function) on $\mathbb{R}^d$ and X a Cox process driven by the shot-noise random field (Møller, 2003)

$$\Lambda(\mathbf{u}) = \rho(\mathbf{u}) \frac{1}{\kappa} \sum_{\mathbf{v} \in \Phi} k(\mathbf{u} - \mathbf{v}), \quad \mathbf{u} \in S \subset \mathbb{R}^d.$$

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Then X is an inhomogeneous Neyman–Scott process on S with the parent process Φ, dispersion kernel k and intensity function $\rho : S \to [0, \infty)$ (Waagepetersen, 2007). The so-called (modified) Thomas process is an important special case where

$$k(\mathbf{u}) = \frac{1}{(2\pi w^2)^{\frac{d}{2}}} \exp\left(-\frac{\|u\|^2}{2\omega^2}\right)$$

is the density function of the multivariate normal distribution $\mathcal{N}_d(\mathbf{0}, \omega^2 I_d)$.

It should be noted that a Neyman–Scott process (Neyman and Scott, 1958) belongs to the class of Poisson cluster processes (Daley and Vere-Jones, 2003, Section 6.3), and, in general, it is not necessarily a shot-noise Cox process (Møller, 2003). In fact, a Neyman–Scott process is a Cox process if the number of points per cluster follows a Poisson distribution (Chiu et al., 2013, Section 5.3). Moreover, there are different approaches to introduce inhomogeneity into Cox processes (Prokešová, 2010). The above considered inhomogeneity is based on the location dependent thinning mechanism (Waagepetersen, 2007), which ensures the second-order intensity-reweighted stationarity of the resulting Neyman–Scott process (Baddeley et al., 2000).

The mth order product density function of X is given by (Møller, 2003)

$$\rho^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m) = \mathbb{E}\big[\Lambda(\mathbf{u}_1) \cdots \Lambda(\mathbf{u}_m)\big], \quad \mathbf{u}_1, \dots, \mathbf{u}_m \in S,\ m \in \mathbb{N}.$$

In particular, $\rho^{(1)}(\mathbf{u}) = \mathbb{E}[\Lambda(\mathbf{u})] = \rho(\mathbf{u})$ is the intensity function of X. For $m \geq 2$, define the intensity-reweighted product densities

$$g^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m) = \frac{\rho^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m)}{\rho(\mathbf{u}_1) \cdots \rho(\mathbf{u}_m)} = \mathbb{E}\left[\prod_{i=1}^{m} \frac{\Lambda(\mathbf{u}_i)}{\rho(\mathbf{u}_i)}\right], \quad \mathbf{u}_1, \dots, \mathbf{u}_m \in S. \tag{1}$$

The pair correlation function

$$g(\mathbf{u}_1, \mathbf{u}_2) = g^{(2)}(\mathbf{u}_1, \mathbf{u}_2) = 1 + \frac{1}{\kappa}\beta^{(1)}(\mathbf{u}_1 - \mathbf{u}_2),$$

is a special case where (Møller and Waagepetersen, 2003)

$$\beta^{(1)}(\mathbf{u}) = \int_{\mathbb{R}^2} k(\mathbf{u} + \mathbf{v}) k(\mathbf{v}) \mathrm{d}\mathbf{v}. \tag{2}$$

The pair correlation function $g(\mathbf{u}_1, \mathbf{u}_2)$ depends on $\mathbf{u}_1$ and $\mathbf{u}_2$ only through the spatial lag $\mathbf{h} = \mathbf{u}_1 - \mathbf{u}_2$; i.e. $g(\mathbf{u}_1, \mathbf{u}_2) = \breve{g}(\mathbf{h})$ where $\breve{g}(\mathbf{h}) = 1 + \beta^{(1)}(\mathbf{h})/\kappa$. Thus, X is a second-order intensity-reweighted stationary process (Baddeley et al., 2000). When X is homogeneous; i.e. $\rho(\mathbf{u}) \equiv \rho$, then $\rho\breve{g}(\mathbf{h}) = \rho\big[1 + \beta^{(1)}(\mathbf{h})/\kappa\big]$ is called the Palm intensity function of the process (see e.g. Ogata and Katsura, 1991; Tanaka, 2013).

For $m \geq 2$ and $1 \leq j \leq m$, let $\mathcal{P}_{j,m}$ denotes the set of all j-partitions of the set $\{1, 2, \dots, m\}$; i.e., for each $\mathcal{C} \in \mathcal{P}_{j,m}$, $\mathcal{C} = \big\{A_1^{(j)}, A_2^{(j)}, \dots, A_j^{(j)}\big\}$ where $A_l^{(j)} \neq \emptyset, A_l^{(j)} \cap A_{l'}^{(j)} = \emptyset$ and $\cup_{l=1}^{j} A_l^{(j)} = \{1, \dots, m\}$ (see Daley and Vere-Jones, 2003, p. 121). If $n(A_l^{(j)})$ denotes the number of elements (cardinality) of $A_l^{(j)}$, then $1 \leq n(A_l^{(j)}) \leq m$ and $\sum_{l=1}^{j} n(A_l^{(j)}) = m$. The cumulant density functions of X, $q^{(l)}(\mathbf{u}_1, \dots, \mathbf{u}_l)$, $l \geq 1$, are related to the product density functions by (see Daley and Vere-Jones, 2003, p. 148)

$$\rho^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m) = g^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m) \prod_{j=1}^{m} \rho(\mathbf{u}_j) = \sum_{j=1}^{m} \sum_{\mathcal{C} \in \mathcal{P}_{j,m}} \left(\prod_{l=1}^{j} q^{\left(n(A_l^{(j)})\right)}(\mathbf{u}_{1l}, \dots, \mathbf{u}_{n(A_l^{(j)})l})\right). \tag{3}$$

For example, $q^{(1)}(\mathbf{u}) = \rho(\mathbf{u})$ and $q^{(2)}(\mathbf{u}_1, \mathbf{u}_2) = \rho(\mathbf{u}_1)\rho(\mathbf{u}_2)\big[g(\mathbf{u}_1, \mathbf{u}_2) - 1\big]$.

3. The higher order product densities

The following Lemma 1 yields a representation formula for the intensity-reweighted product densities of the Neyman–Scott process X in Section 2. The same expression can be obtained by using the probability generating functional of X (see Daley and Vere-Jones, 2003, p. 146) to find the cumulant density functions and substitute them in (3).

Lemma 1. *For $m \geq 2$,*

$$g^{(m)}(\mathbf{u}_1, \dots, \mathbf{u}_m) = 1 + \sum_{j=1}^{m-1} \frac{1}{\kappa^{m-j}} \sum_{\mathcal{C} \in \mathcal{P}_{j,m}} \left(\prod_{l=1}^{j} \int_{\mathbb{R}^d} \left[\prod_{i_l \in A_l^{(j)}} k(\mathbf{u}_{i_l} - \mathbf{v})\right] \mathrm{d}\mathbf{v}\right). \tag{4}$$

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