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# On renewal increasing mean residual life distributions: An age replacement model with hypothesis testing application

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ABSTRACT

We study the life distribution of an operating device through the notion of mean residual life. The device is experiencing a random number of shocks governed by a homogeneous Poisson process, and a new U-statistic test procedure is introduced to test the hypothesis of that the life is exponentially distributed against the alternative that the life distribution is the life distribution has renewal increasing mean residual property.

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## 1. Introduction

Let $X$ denote a nonnegative random variable with a continuous life distribution function $F$ and finite mean $\mu = \int_0^\infty \bar{F}(x)dx$. The mean residual life function at time $t$ is defined as

$$m(t) = E(X - t|X > t) = \frac{1}{\bar{F}(t)} \int_t^\infty \bar{F}(u)\, du.$$

By addressing the behavior of $m(t)$ with respect to $t$, one may realize the optimal times for the schemes of replacement before failure occurs (Kayid et al., 2013).

In the literature, when the lifetime of an operating device is not affected by any tempered events, various nonparametric procedures for testing exponentiality against monotonicity properties of the MRL function have been proposed and studied. The reader is referred to Ahmad (1992), Lim and Park (1993), Lim and Park (1997), Abu-Youssef (2002, 2004), Asadi (2005), Ahmad and Sepehrifar (2009), Asha and Nair (2010), and Kayid and Izadkhah (2014) among others. In some situations, the device experiences a number of shocks governed by a homogeneous Poisson process. Due to these tempered events, the lifetime of such a device becomes shortened or prolonged. In the past decade, many properties of such life distributions have been extensively investigated. See Abouammoh et al. (1993), Li and Xu (2008), Ahmad and Mugdadi (2010), and Izadkhah and Kayid (2013) among others.

The goal of this paper is to study age replacement models through the remaining lifetime of a device operating in a more realistic environment. Such a device fails by physical deterioration caused from some damage. In the latter case, a device

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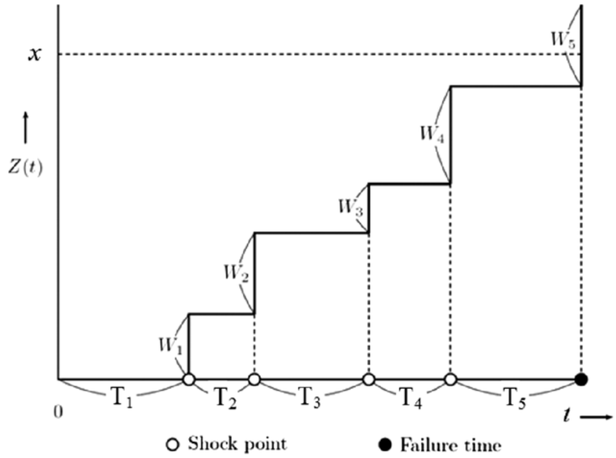


**Fig. 1.** Process for a cumulative life-damage model. The random variables $\{T_j : j = 1, 2, \ldots\}$ denote the sequence of time intervals between successive shocks, which are independent of the i.i.d. sequence $\{W_i : i = 1, 2, \ldots\}$; the amount of life-damage produced by the $i$th shock, with $W_0 \equiv 0$.

fails when the total damage due to tempered events exceeds a critical level. Such damage models may apply to the actual units that are working in industry, service, information, computers, etc.

## 2. Basic definitions and properties

Consider a unit which is subjected to successive shocks where each shock causes some damages. Let the random variable $X$ be the lifetime of such a device with survival function $\bar{F}(t) = P\{X \geq t\}$. Let the random variable $N(t)$ denote the total number of shocks up to time $t$ with probability mass function $P\{N(t) = j\} = F^{(j)}(t) - F^{(j+1)}(t)$, $j = 0, 1, 2, \ldots$. Let, also, the random variable $W_j$, $j = 0, 1, 2, \ldots$ be the amount of hidden lifetime absorbed by the $j$th shock ($W_0 \equiv 0$), with common distribution $G(x) = P\{W_j \leq x\}$. The total cumulative life-damage up to time $t$ is defined as $Z(t) = \sum_{i=0}^{N(t)} W_i$ with the cumulative distribution function $Q(x) = P\{Z(t) \leq x\} = \sum_{j=0}^{\infty} G^{(j)}(x)\left[F^{(j)}(t) - F^{(j+1)}(t)\right]$. See Glynn and Whitt (1993) and Roginsky (1994). It is assumed that the unit fails when the total life-damage exceeds a pre-specified level $x > 0$ (Fig. 1). The failure level $x$ is statistically estimated and is already a known value. Let $X^* = X - Z(t)$ be the residual lifetime of an operating device in a service with total life-damage $Z(t)$. In practice, the realizations of $X^*$ are recorded.

The mean residual lifetime function (MRL) is a well-known reliability measure in the age replacement models. Consider a device subjected to $N(t)$ number of shocks up to time $t$. Given that such a device is in an operating situation at time instant $t$ after installation, the MRL function of $X^*$ is defined by $m^*(t) = E\left[X_t^*\right] = E[X - Z(t) - t | X - Z(t) \geq t]$. Note that the total life-damage has not exceeded the threshold level $x$. We assume that random variables $X$ and $Z(t)$ are independent. Let us present some definitions and basic properties that will be used in the sequel.

**Definition 2.1.** The mean residual life of a device under shock models ($\mathrm{MRL}_{\mathrm{shock}}$) at time $t$, is defined by $m^*(t) = \frac{\int_t^\infty \bar{v}(z)dz}{\bar{v}(t)}$, where $\bar{v}(z) = \int_0^x \bar{F}(z + w)\, dQ(w)$.

**Definition 2.2.** The lifetime random variable $X$ is said to be renewal increasing mean residual life under shock models ($\mathrm{RIMRL}_{\mathrm{shock}}$), if $m^*(t)$ is a non-decreasing function in $t \geq 0$, whenever the derivative exits.

**Corollary 2.1.** *The distribution $F$ belongs to* $\mathrm{RIMRL}_{\mathrm{shock}}$ *class if*

$$\frac{\partial[m^*(t)]}{\partial t} = \frac{-(\bar{v}(t))^2 + \int_0^x f(t + w)\, dQ(w) \int_t^\infty \bar{v}(z)\, dz}{(\bar{v}(t))^2} \geq 0,$$

*i.e.,*

$$(\bar{v}(t))^2 \leq \int_0^x f(t + w)\, dQ(w) \int_t^\infty \bar{v}(z)\, dz. \tag{1}$$

## 3. Moment inequality

Through this section, it is assumed that all moments exist and are finite. The following lemma is an extension of the mean value theorem (Zwillinger, 2014).