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Precise large deviation results for sums of sub-exponential claims in a size-dependent renewal risk model

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ABSTRACT

This paper deals with a size-dependent renewal risk model in which claim sizes and inter-occurrence times correspondingly form a sequence of independent and identically distributed random pairs, with each pair obeying a dependence structure described via the conditional distribution of the inter-occurrence time given the subsequent claim size being large. The impact of this dependence structure on the tail behavior of aggregated claims is investigated and then a precise large deviation formula for the aggregate amount of sub-exponential claims is obtained.

1. Introduction

Ever since Andersen (1957) extended the Cramèr–Lundberg model (or classical compound Poisson risk model), the renewal risk model has over the years played an important role in the modern risk theory. In the relative literature, most research work allowed claim inter-occurrence times to have arbitrary distribution functions but maintained the assumption of independence.

Consider a continuous-time renewal risk model, in which claim sizes Z_k, $k \in \mathbb{N}$, occur at successive renewal epochs with inter-occurrence times θ_k, $k \in \mathbb{N}$. Assume that (Z_k, θ_k), $k \in \mathbb{N}$, form a sequence of independent and identically distributed (i.i.d.) copies of a generic random pair (Z, θ) with marginal distribution functions $F = 1-\overline{F}$ on $[0, \infty)$ and G on $[0, \infty)$, which has finite means $a = \mathrm{E}Z$, $1/\lambda = \mathrm{E}\theta$ and finite second moments $\mathrm{E}Z^2 < \infty$, $\mathrm{E}\theta^2 < \infty$. Denote by $T_k = \sum_{i=1}^{k} \theta_i$, $k = 1, 2, 3, \ldots$ the claim occurrence times, with $T_0 = 0$. Then the number of claims by given time t is $N_t = \#\{k \in \mathbb{N} : T_k \le t\}$ with a mean function $\lambda(t) = \mathrm{E}N_t$ satisfying $\lambda(t) \sim \lambda t$ as $t \to \infty$, which forms an ordinary renewal counting process. The aggregate claims up to time t (≥ 0) are given by compound sum of the form

$$S_t = \sum_{k=1}^{N_t} Z_i, \quad t \ge 0, \tag{1.1}$$

where, by convention, the cardinality of the empty set of index is zero. All random variables are assumed to be non-degenerated at zero.

Literature on precise large deviation for the standard renewal risk model has been extensively studied in the field of insurance. Klüppelberg and Mikosch (1997), Kaas and Tang (2005), Ng et al. (2004) and Tang et al. (2001), among many others have studied precise large deviations for different tailed random sums. Baltrūnas et al. (2008) studied the renewal risk

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model with sub-exponential claim sizes and precise large deviation results for random sums assuming that claim sizes and inter-occurrence times are independent. However, such independence can be unreasonable in reality but good for mathematical tractability. In practical terms, when the deductible on each loss is raised, claim sizes would naturally decrease thereby increasing the inter-occurrence time because small losses will be retained by the insured. Hence, in order to meet the needs of practice, we release the assumption of independence between the claim size Z_k and inter-occurrence time θ_k as shown in this paper.

During the last decade, some dependence structures have been proposed to relax these independence assumptions. Pioneered by Albrecher and Teugels (2006), many scholars have investigated related problems of an extension to the renewal risk model in which the components of the random pair (Z_k, θ_k), $k = 1, 2, 3, \ldots$ are dependent. Badescu et al. (2009) posited that (Z, θ) has a bivariate phase type distribution and they made use of the existing connection among risk processes and fluid flows. Other non-standard renewal risk model extensions within the above framework can be seen in Boudreault et al. (2006), Cossette et al. (2008), Asimit and Badescu (2010), Li et al. (2010) and Chen and Yuen (2012).

In this paper, we employ the same dependence structure as proposed by Chen and Yuen (2012) for the generic random pair (Z, θ). Precisely, we make the following assumption:

Assumption 1. There is a nonnegative random variable θ^* and some $x_0 > 0$ such that it holds for all $x > x_0$ and $t \in [0, \infty)$ that

$$\mathrm{P}(\theta > t | Z > x) \leq \mathrm{P}(\theta^* > t). \tag{1.2}$$

As pointed out by Chen and Yuen (2012), Assumption 1 states that θ conditional on the event $(Z > u)$ is stochastically bounded by θ^* for all large $u > 0$ meaning that Z becoming large does not drag θ to infinity. The risk model (1.1) in which (1.2) holds is termed by Chen and Yuen (2012) as a size-dependent renewal risk model.

We investigate precise large deviation results for $\{S_t, t \geq 0\}$ under mild assumptions that claim sizes follow a sub-exponential distribution function and the generic pair (Z, θ) are dependent. The class of sub-exponential distribution is a natural and useful class of heavy-tailed distributions. By definition, a distribution $F(x)$ on $[0, \infty)$ is sub-exponential and represented by $F \in \mathcal{S}$ if the tail $\overline{F}(x) = 1 - F(x) > 0$ for all $x \geq 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{F^{*n}}(x)}{\overline{F}(x)} = n$$

holds for all (or, for some) $n = 2, 3, \ldots$, where F^{*n} is the n-fold convolution of F. For applications of precise large deviation results in the fields of finance and insurance, the reader is referred to Mikosch and Nagaev (1998), Klüppelberg and Mikosch (1997), among others.

There is an important subclass of $\mathcal{S}$, denoted by $\mathcal{C}$ which consists of distribution functions with consistent variation. A distribution function $F(x)$ with support on $[0, \infty)$ is said belonging to class $\mathcal{C}$ if

$$\lim_{y\searrow 1} \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = \lim_{y\nearrow 1} \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1.$$

When $F \in \mathcal{C}$, with Assumption 1, Chen and Yuen (2012) extended the study of precise large deviations to the case allowing (both positive and negative) dependence between claims and their inter-occurrence times. In addition to Assumption 1, they assume that, $\mathrm{E}Z = a \in (0, \infty)$ and $\mathrm{E}\theta = 1/\lambda \in (0, \infty)$, and they obtain, for arbitrarily given $\gamma > 0$, it holds uniformly for all $x \geq \gamma t$ that

$$\mathrm{P}(S_t - a\lambda t > x) \sim \lambda t \overline{F}(x), \quad t \to \infty, \tag{1.3}$$

that is,

$$\lim_{t\to\infty} \sup_{x\geq\gamma t} \left| \frac{\mathrm{P}(S_t - a\lambda t > x)}{\lambda t \overline{F}(x)} - 1 \right| = 0.$$

Following their work, we study the size-dependent renewal risk model, but relax their assumption on the distribution F with $F \in \mathcal{S}$. We prove that, for every positive $u > 0$, the relation

$$\mathrm{P}(S_t > x + (a + u)\lambda t) \sim \lambda t \overline{F}(x + u\lambda t), \quad x \to \infty \tag{1.4}$$

holds uniformly for all $t \in [f(x), \gamma x / Q(x)]$, where $f(x)$ is an arbitrary infinitely increasing function, $\gamma > 0$ is an arbitrary positive constant and $Q(x) = -\ln \overline{F}(x)$, $x \in \mathbb{R}_+$ is the hazard function of distribution F. In our model, $u\lambda t$ is the part of premium used to pay for operating expenses of an insurance company. Hence, $u > 0$ can be an arbitrary positive constant.

Compared with relation (1.3), relation (1.4) with respect to $x \to \infty$ is more natural for studying the asymptotics of the finite time ruin probabilities, where the initial capital of an insurance company, x, tends to infinity. Related discussions can be found in Baltrūnas et al. (2008) in which the same relation as (1.4) was investigated, but maintained the assumption of independence between Z and θ. However, allowing dependence between claims and their inter-occurrence times would be more realistic for evaluating some risk measures such as conditional tail expectation of the aggregate amount of claims from a large insurance portfolio.