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Some asymptotic results of the ruin probabilities in a two-dimensional renewal risk model with some strongly subexponential claims

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ABSTRACT

In this paper, a two-dimensional renewal risk model of insurance business with some strongly subexponential claim sizes is considered. Three types of the finite-time ruin probabilities under this model are discussed. We obtain the asymptotic upper and lower bounds for one type, and the asymptotic formulas for the others, which hold uniformly in a corresponding region, respectively.

1. Introduction

The renewal risk model has attracted enormous attention in the insurance and applied probability literature since it was introduced by Andersen (1957) half a century ago. In this model, the claim sizes $\{X_i, i \geq 1\}$ form a sequence of i.i.d. nonnegative random variables with a common distribution function $F(x)$ and a finite mean $EX_1 = b$, and the inter-occurrence times $\{\theta_i, i \geq 1\}$ are i.i.d. nonnegative random variables with finite mean $E\theta_1 = 1/\lambda$ and the finite second moment $E\theta_1^2$. We assume that $\{\theta_i, i \geq 1\}$ are mutually independent of $\{X_i, i \geq 1\}$. The random variables $T_n = \sum_{i=1}^{n} \theta_i, n = 1, 2, \ldots$ constitute a renewal counting process $N(t) = \sharp\{n = 1, 2, \ldots : T_n \in (0, t]\}$ with a mean function $\lambda(t) = EN(t)$, where $\lambda(t) \sim \lambda t$ as $t \to \infty$. Furthermore, by the well-known approximation $\operatorname{Var} N(t) = O(t)$, it holds that $EN^2(t) = \operatorname{Var} N(t) + [EN(t)]^2 \sim (\lambda t)^2$ (see, e.g., Section 2.5 of Embrechts et al., 1997).

The surplus process of the insurance company is given by

$$R(t) = x + ct - \sum_{i=1}^{N(t)} X_i, \quad t \geq 0,$$

where $x \geq 0$ is the initial surplus, $c > 0$ is the constant premium rate and $\sum_{i=1}^{0} X_i = 0$ by convention.

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Let

$$\Psi(x,t)=P\left(\inf_{0\le s\le t}R(s)<0\,|R(0)=x\right)$$

be a ruin probability up to time t and it is natural to assume that the following safety loading condition: $\mu = cE\theta_1 - EX_1 = c/\lambda - b > 0$.

Leipus and Šiaulys (2007) investigated asymptotic behaviour of the finite-time ruin probability $\Psi(x,t)$ in the case of subexponential claim sizes, and proved that the relation

$$\Psi(x,t)\sim\frac{1}{\mu}\int_x^{x+\mu\lambda t}\overline{F}(u)du,\quad x\to\infty \tag{1.1}$$

holds uniformly for all $t\in[f(x),\gamma x]$, where $f(x)$ is an arbitrary infinitely increasing function, γ is an arbitrary positive constant and $\overline{F}=1-F$.

Recently, the dependent classes of insurance business have been extensively investigated in the literature due to their practical importance. Focusing on multivariate regularly varying random walks, Hult et al. (2005) initially studied the ruin probability for multi-dimensional heavy tailed process and provided sharp asymptotics for general ruin boundaries. Yuen et al. (2006) discussed various methods for evaluation of ruin probability in two dependent classes of insurance business. Li et al. (2007) added a diffusion component in the two-dimensional compound Poisson risk model, and obtained some estimates of the finite time ruin probability and the ultimate ruin probability, respectively. Chen et al. (2011) extended the one dimensional result in Tang (2004) to a two-dimensional renewal risk model for claims with consistently varying tails.

It is these results that motivate our study. In this paper, on the one hand, comparing with Leipus and Šiaulys (2007), we extend (1.1) to a two-dimensional renewal risk model; on the other hand, comparing with Chen et al. (2011), we extend the consistently varying tails to some strongly subexponential tails.

The rest of the paper is organized as follows: In Section 2, we present some related notations and useful lemmas. The main results are given in Section 3. Finally, the proofs of the main results are presented in Sections 4–6, respectively.

2. Preliminaries

We say X (or its distribution F) is heavy tailed if it has no exponential moments. An important heavy tailed subclass is the subexponential class $\mathcal{S}$. A distribution function F with support on $[0,\infty)$ belongs to $\mathcal{S}$, if the tail $\overline{F}=1-F$ satisfies equality

$$\lim_{u\to\infty}\overline{F*F}(u)/\overline{F}(u)=2,$$

where $F*F$ denotes the Stieltjes convolution of F with itself.

Korshunov (2002) introduced strongly subexponential class, denoted as $F\in\mathcal{S}_*$, if it has finite mean and d.f. F_u defined by equality

$$\overline{F}_u(x)=\begin{cases}\min\left\{1,\int_x^{x+u}\overline{F}(y)dy\right\} & \text{if } x\ge 0,\\ 1 & \text{if } x<0,\end{cases}$$

satisfies the relation

$$\lim_{x\to\infty}\overline{F_u*F_u}(x)/\overline{F_u}(x)=2$$

uniformly for $u\in[1,\infty)$.

A d.f. F of a nonnegative random variable belongs to the class $\mathcal{S}^*$ if it has finite mean and

$$\lim_{x\to\infty}\int_0^x\frac{\overline{F}(x-u)}{\overline{F}(x)}\overline{F}(u)du=2\int_0^\infty\overline{F}(u)du.$$

Note that $\mathcal{S}^*\subset\mathcal{S}$ (see Klüppelberg, 1988). According to Lemma 9 of Denisov et al. (2004), it follows that $F\in\mathcal{S}^*$ implies $F_u\in\mathcal{S}$, i.e. $\mathcal{S}^*\subset\mathcal{S}_*$.

In Baltrūnas et al. (2008), $Q(u)=-\log\overline{F}(u)$, $u\in\mathcal{R}_+$ denotes the hazard function of distribution F. They also assumed that there exists a nonnegative function $q:\mathcal{R}_+\to\mathcal{R}_+$ such that $Q(u)=\int_0^u q(v)dv$, $u\in\mathcal{R}_+$. The function q is called the hazard rate of d.f. F. Denote by

$$r:=\limsup_{u\to\infty}uq(u)/Q(u)$$

a hazard ratio index. Note that if the hazard ratio index satisfies $r<1$ then F is subexponential (see Lemma 3.8(a) in Baltrūnas et al., 2004).

For giving the main results of this paper, we need the following assumptions in the model, where Assumptions A_1–A_5 come from Assumptions H_1, H_2, A–C in Leipus and Šiaulys (2007), and Assumption A_6 comes from Assumptions H_1 and A