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Testing for the cointegration rank in threshold cointegrated systems with multiple cointegrating relationships

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ABSTRACT

This paper is concerned with the estimation and inference for a threshold VECM with more than one cointegrating relation thus extending the literature in this context which has mostly considered only one cointegrating relationship so far. We then go on to develop an appropriate test for the number of cointegrating relationships in a threshold VECM. Asymptotic distributions of our test statistics are derived and tabulated. Finite sample performance of the proposed testing procedure is investigated.

1. Introduction

Recent years have seen a surge of interest in the examination of the different possible ways to take into account non-linear adjustments to equilibrium. Balke and Fomby [4] were the first authors to introduce the idea of threshold cointegration which specifies a threshold autoregressive (TAR) process for the equilibrium error. The TAR models, first investigated by Tong [38], are particularly popular in empirical applications because of their capacity to adequately represent observed behavior of time series in terms of persistence and asymmetry. In addition, the statistical properties of

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TAR processes have been extensively studied in the literature outside of cointegration theory (see [39,40,7,17,18,12,5] among numerous others) and therefore many results are available for conducting inference.

Subsequent to the study by Balke and Fomby [4], several tests have been proposed in the literature on threshold cointegration. For example, Enders and Siklos [11] use self-exciting TAR (hereafter SETAR) as well as Momentum TAR (M-TAR) processes to describe the disequilibrium and propose a methodology to test for cointegration but without a real asymptotic test theory. Lo and Zivot [27] extend the work of Enders and Siklos and test threshold adjustment in international relative prices. Kapetanios et al. [26] have dealt with a disequilibrium which follows an exponential smooth transition process and have proposed a non-linear residual-based test for cointegration. Hansen and Seo [20] and Gonzalo and Pitarakis [16] provide statistics and asymptotic theory for testing the existence of a threshold effect in the error correction model. Gonzalo and Pitarakis [16] also investigate the stochastic properties of the error correction model with threshold effect and in particular the stability conditions. In [36] the equilibrium error is specified as a SETAR process a non-linear cointegration test. Kapetanios and Shin [25] propose a test similar to that of Seo [36] in a three-regime SETAR model where the corridor regime follows a random walk. More recently Seo [37] derives the asymptotic properties of the long-run, short-run and the threshold parameter estimators in regime switching error correction models. Note that two pitfalls are combined when one wishes to test for a unit root in a threshold model in which the transition variable is the lagged dependent variable itself. In addition to the well known issue of non-identification of the threshold parameter under the null hypothesis inherent in TAR models [17,18], the transition variable is also nonstationary under the null. Thus, although the model of Caner and Hansen [5] and Gonzalo and Pitarakis [16] covers a large set of processes, it does not include this particular case considered in [36] and in [25]. However, as discussed in [5], one could always take the lagged variation of the threshold variable in order to ensure its stationarity. Enders and Siklos [11] note that this choice could often be suitable in economic models. This paper is mainly concerned with the development of a test for the number of cointegrating relationships in a threshold framework. We propose an LM-type test procedure along the lines of Lütkepohl and Saikkonen [29] and derive the asymptotic distribution of our statistic. This extends the results of Hansen and Seo [20] who, as most of the other authors cited above, consider only one cointegrating relationship. Although determining the number of cointegrating relationships for a set of integrated variables and estimating them has been an important area of research in standard cointegration theory (one can cite Johansen [22,23], Phillips [33], and Phillips and Durlauf [34] among others), there are few developments in the threshold cointegrated context. To our knowledge, Gonzalo and Pitarakis [16] is the only study to propose a test for cointegrating rank in a threshold vector error correction model (TVECM) by directly estimating the unknown ranks of the coefficient matrices using a model selection approach introduced by Gonzalo and Pitarakis [13,14]. Whilst the simplicity of their methodology is attractive, the estimation depends on a penalty parameter for which the choice is somewhat tricky since no asymptotic minimax universal value is available. We propose an alternative (and more traditional) route based on the Lagrange Multiplier (LM)-test. We provide the asymptotic distributions of our estimators and test statistics, Monte-Carlo experiments are conducted and show our testing procedure performs well in finite sample context.

The remainder of this paper is organized as follows. Section 2 is devoted to the model formulation and the required assumptions for the asymptotic theory. The analytic solution for long-run estimator is also derived as well as its limiting distribution. Inference is discussed in Section 3, in particular, a testing procedure for the threshold cointegrating rank is proposed. The asymptotic distributions of the test statistics are derived and tabulated. In Section 4 Monte-Carlo experiments are performed to investigate the finite sample performance of the proposed testing procedure. The last section summarizes our work and concludes.

The following general notation is used. A $(p \times p)$ identity matrix is denoted by I_p. Whenever the dimension needs to be specified, we will denote a $(p \times m)$ matrix A as $A_{(p\times m)}$. The projection matrix associated with a $(p \times m)$ matrix A will be denoted as P_A, the orthogonal complement of A as $A_\perp$ which is a $(p \times p - m)$ matrix of full column rank such that $A'A_\perp = 0$. $\mathbb{I}_{\{\cdot\}}$ will design the indicator function. We will use $I(1)$ and $I(0)$ to represent time series that are integrated of order 1 and 0, respectively. Throughout the paper, integrals are taken over the unit interval and we use