Statistical Methodology 26 (2015) 103–123

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Estimating the parameters of a seasonal Markov-modulated Poisson process

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A R T I C L E I N F O

Article history:
Received 20 April 2014
Received in revised form
16 December 2014
Accepted 19 April 2015
Available online 28 April 2015

MSC:
primary 62M05
62F12
secondary 60F05
60F15

Keywords:
Markov-modulated Poisson process
Seasonality
Split-time likelihood
Strong consistency
Asymptotic normality

A B S T R A C T

Motivated by seasonality and regime-switching features of some insurance claim counting processes, we study the statistical analysis of a Markov-modulated Poisson process featuring seasonality. We prove the strong consistency and the asymptotic normality of a maximum split-time likelihood estimator of the parameters of this model, and present an algorithm to compute it in practice. The method is illustrated on a small simulation study and a real data analysis.

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1. Introduction

It is often the case that the insurance claim frequency is impacted by environment variables. For instance, flood risk is higher in a period of frequent heavy rains, and fire risk is more intense when the weather is particularly dry. Such environment variables may be hidden to some extent to the

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http://dx.doi.org/10.1016/j.stamet.2015.04.003
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practitioner: for instance, it is now accepted that the probabilities of severe floods in Australia, strong snowstorms in North America or hurricanes on the East Coast of the United States increase during La Niña episodes (see Neumann et al. [17], Cole and Pfaff [5], Parisi and Lund [20] and Landreneau [10]). This is now taken seriously by most reinsurers as well as Lloyd's and the UK Met Office [13]. However, observing and understanding the role of those variables is not easy, which makes it realistic to consider these variables as unobserved so far.

To take such a dependency into account, one may for instance assume that the underlying environment process is a Markov process J in continuous time and that in each state of J, the claim counting process N is a Poisson process. The resulting bivariate process (J, N) is then called a Markov-Modulated Poisson Process (MMPP). MMPPs have been used in different fields during the past forty years, in particular in data traffic systems (see *e.g.* Salvador et al. [27]), for ATM sources (see Kesidis et al. [9]), in manufacturing systems (see *e.g.* Ching et al. [4]) or even in ecology (see for example Skaug [29] for applications of MMPPs to clustered line transect data). The MMPP cookbook by Fischer and Meier-Hellstern [6] sums up the main results and ideas that were behind the rise of MMPP applications. The idea of considering Markov modulation in insurance was first introduced by Asmussen [2]; the obtained model can capture the fact that the insurance claim frequency may be modified if climatic, political or economic factors change. Such a model has gained considerable attention recently: see for instance Lu and Li [15], Ng and Yang [18], Zhu and Yang [33] and Wei et al. [32]. The parameters of an MMPP are often estimated using a Maximum Likelihood Estimator (MLE), whose consistency was proved in Rydén [23]. Various methods have been suggested to compute the MLE; a standard tool is the Expectation–Maximization (EM) algorithm, see Rydén [25] for the implementation of this procedure for the estimation of the parameters of an MMPP. We finally mention that in a recent paper, Guillou et al. [7] introduced a new MMPP-driven loss process in insurance with several lines of business, showed the strong consistency of the MLE and fitted their model to real sets of insurance data using an adaptation of the EM algorithm.

Of course, once the MMPP model is fitted, it is possible to use Bayesian techniques to determine probabilities to be in each state and consequently the average number of events during the next period (see *e.g.* Scott [28]). If external information is present, then it is possible to enrich this Bayesian estimation. This is for example the case for some long-tailed non-life insurance businesses, where indices of sectorial inflation can provide useful information. For reinsurance cycles, large claims that may cause a cycle phase change as well as other aspects of competition or adverse development of reserves can sometimes be plugged into the Bayesian estimation process. For some other risks however, even if we feel that a phenomenon might have an impact on the claim frequency, it may be very hard to come up with a measure of such a phenomenon (for instance, the El Niño–La Niña phenomena). In that case our non-Bayesian framework is of interest for actuarial risk assessment.

Furthermore, many examples of practical applications in insurance display some sort of seasonal variation. For example, theft in garages are more frequent before Christmas as people tend to store Christmas gifts there, fire risk is more intense in the summer, and hurricanes occur mostly between June and November on the East Coast of the United States. These random, cyclic factors and their impact on insurance risk, which need to be taken into account to carry out a proper regime switching analysis, are yet to be understood and forecasted. In an inhomogeneous context with deterministic intensity function, Lu and Garrido [14] have fitted double-periodic Poisson intensity rates to hurricane data, for particular parametric forms (like double-beta and sine-beta intensities) to hurricane data. Helmers et al. [8] have provided an in-depth theoretical statistical analysis of such doubly periodic intensities. We aim at carrying out a theoretical statistical analysis in a stochastic intensity framework with seasonality.

An important aspect of pricing in non-life insurance concerns segmentation: thanks to generalized linear models or more sophisticated techniques, the insurer takes into account explanatory variables to adapt the price of the contract and avoid adverse selection (see Ohlsson and Johansson [19]). Besides, individual ratemaking is updated thanks to credibility adjustments in order to take into account the claim history of each policyholder or contract (see Bühlmann and Gisler [3]). Our approach is only operational and interesting at the aggregated risk management level, and it would be very challenging to try to combine it with regression techniques, from a theoretical point of view as well as the practical point of view since a very large number of data points would be needed to ensure that

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