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A fluctuation limit theorem for a critical branching process with dependent immigration

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ABSTRACT

We consider a critical branching process with immigration controlled by a sequence of random variables $\{\xi_k, k \geq 1\}$, and prove a fluctuation limit theorem when $\{\xi_k\}$ is $(N-1)$-dependent, improving the result in previous literatures.

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1. Introduction

Suppose $\{X_{ni}, n, i \geq 1\}$ and $\{\xi_n, n \geq 1\}$ are two sequences of non-negative integer-valued random variables, and $\{X_{ni},\ n, i \geq 1\}$ are independent and identically distributed (i.i.d.). Define $\{Z(n)\}$ recursively as

$$Z(n) = \sum_{i=1}^{Z(n-1)} X_{ni} + \xi_n, \quad n \geq 1, \quad Z(0) = 0. \tag{1.1}$$

Suppose $A := EX_{ni} = 1$. $\{Z(n) : n \geq 0\}$ is called a critical branching process with immigration. In this paper, we suppose $B := \mathrm{Var}(X_{ni}) \in (0, \infty)$.

There have been many research works on the limit theorems of branching processes with generalized immigration. For instance, Nagaev (1975) considered a stationary immigration process $\{\xi_n\}_{n=1}^{\infty}$ in wide sense, namely, $\mathrm{Cov}(\xi_1, \xi_n) \to 0$ as $n \to \infty$, and obtain the limit distribution of total population. Asadullin and Nagaev (1982) studied this problem under more general situation that there exists a random variable ξ, such that $n^{-1}E|\sum_{i=1}^{n}(\xi_i - \xi)| \to 0$ as $n \to \infty$. They also studied the case of continuous time. Rahimov (1995) provided a variety of results and references related to generalization of the immigration process. In Chapter II and Chapter III therein, most general immigration processes given by an arbitrary point process are discussed. Recently, Rahimov (2007) considered a critical discrete-time branching process with immigration defined by (1.1). Under the assumption that $\{X_{ni}, n, i \geq 1\}$ and $\{\xi_n, n \geq 1\}$ are independent and the immigration process $\{\xi_k, k \geq 1\}$ is a sequence of independent random variables, he proved functional limit theorems for the normalized processes of $\{Z(n)\}$.

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In this paper, we shall study the fluctuation limit of $\{Z(n), n \geq 0\}$ defined by (1.1) under the following assumption:

(H) The two sequences $\{X_{ni}, n, i \geq 1\}$ and $\{\xi_n, n \geq 1\}$ are independent with each other. There exists $N \geq 2$, such that ξ_i and ξ_j are independent for $|i-j| \geq N$.

Under this condition, $\{\xi_n, n \geq 1\}$ is $(N-1)$-dependent. Roughly speaking, two immigration random variables are dependent only when they are in neighboring generations, and this dependence disappears when the time interval between two immigration comings is larger than $N-1$. See Diananda and Bartlett (1951) and Watson (1954) for more background and applications in statistics.

The approach we use in the proofs comes from Rahimov (2007), which was introduced by Ispàny et al. (2003, 2005). However, we have to pay more attention to the changes caused by the non-independence of immigration and need more precise analysis and estimation.

The remainder of the paper is organized as follows. The main result and an example are given in Section 2. Section 3 contains some preliminary lemmas and their proofs. Section 4 is devoted to the proof of the main theorem. Finally, an application in parameter estimation is given in Section 5.

2. Functional limit theorem

For each $n \geq 0$, let $\mathscr{F}(n)$ be the σ-algebra generated by $\{Z(k),\ k = 0, 1, \dots, n\}$. Denote $\alpha(n) = E\xi_n < \infty$ and $\beta(n) = \mathrm{Var}(\xi_n)$. For each $n \geq 1$, suppose $\alpha(n)$ and $\beta(n)$ are regularly varying functions as $n \to \infty$, i.e.,

$$\alpha(n) = n^{\alpha} L_{\alpha}(n), \qquad \beta(n) = n^{\beta} L_{\beta}(n),$$

where $\alpha, \beta \geq 0$, and $L_\alpha(n)$ and $L_\beta(n)$ are slowly varying functions as $n \to \infty$. If a sequence $\{a_n, n \geq 1\}$ is regularly varying with exponent ρ, we write $\{a_n\} \in R_\rho$. So $\alpha(n) \in R_\alpha$ and $\beta(n) \in R_\beta$. Denote $A(n) = EZ(n)$ and $B^2(n) = \mathrm{Var}(Z(n))$. By (1.1) and recurrence, for $n \geq 1$,

$$A(n) = E\Big[\sum_{i=1}^{Z(n-1)} X_{ni}\Big] + E\xi_n = EZ(n-1) + \alpha(n) = \cdots = \sum_{k=1}^{n} \alpha(k),$$

and

$$B^2(n) = BA(n-1) + \mathrm{Var}Z(n-1) + \beta(n) + 2\mathrm{Cov}(Z(n-1), \xi_n).$$

Note that

$$\begin{aligned}
\mathrm{Cov}(Z(n-1), \xi_n) &= E[(Z(n-1) - EZ(n-1))(\xi_n - E\xi_n)] \\
&= E\left[\left[\sum_{i=1}^{Z(n-2)} X_{n-1,i} - EZ(n-2) + \xi_{n-1} - \alpha(n-1)\right](\xi_n - E\xi_n)\right] \\
&= \mathrm{Cov}(Z(n-2), \xi_n) + \mathrm{Cov}(\xi_{n-1}, \xi_n) = \cdots = \sum_{i=1}^{n-1} \mathrm{Cov}(\xi_i, \xi_n).
\end{aligned}$$

By iteration, we have

$$B^2(n) = \Delta^2(n) + \sigma^2(n) + 2\sum_{k=2}^{n}\sum_{i=1}^{k-1} \mathrm{Cov}(\xi_i, \xi_k),$$

where

$$\Delta^2(n) = B\sum_{k=1}^{n-1} A(k), \qquad \sigma^2(n) = \sum_{k=1}^{n} \beta(n). \tag{2.1}$$

For $t \geq 0$, define

$$Y_n(t) = \frac{Z([nt]) - A([nt])}{B(n)}, \quad n \geq 1. \tag{2.2}$$

where $[nt]$ is the integer part of nt.

In this paper, '$\stackrel{D}{\to}$', '$\stackrel{d}{\to}$' and '$\stackrel{P}{\to}$' denote the convergence of random functions in Skorokhod space, the convergence of random variables in distribution, and convergence in probability, respectively.

The following is our main theorem of this paper:

Theorem 2.1. *Suppose condition* (H) *is satisfied. If* $\alpha(n) \to \infty$ *and* $\beta(n) = o(n\alpha(n))$ *as* $n \to \infty$*, then*

$$Y_n(t) \stackrel{D}{\to} W(t^{\alpha+2})$$

in Skorokhod space $D(\mathbb{R}_+, \mathbb{R})$ *as* $n \to \infty$.

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