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Some properties of stochastic volatility model that are induced by its volatility sequence

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ABSTRACT

In this paper, we consider a heavy-tailed stochastic volatility model $X_t = \sigma_t Z_t$, $t \in \mathbb{Z}$, where the volatility sequence (σ_t) and the iid noise sequence (Z_t) are assumed to be independent, (σ_t) is regularly varying with index $\alpha > 0$, and the Z_t's to have moments of order less than $\alpha/2$. Here, we prove that, under certain conditions, the stochastic volatility model inherits the anti-clustering condition of (X_t) from the volatility sequence (σ_t). Next, we consider a stochastic volatility model in which (σ_t) is an exponential AR(2) process with regularly varying marginals and show that this model satisfies the regular variation, mixing and anti-clustering conditions in Davis and Hsing (1995).

1. Introduction

The *stochastic volatility model*

$$X_t = \sigma_t Z_t, \quad t \in \mathbb{Z}, \tag{1.1}$$

has attracted considerable attention in the financial time series literature. Here, the *volatility sequence* (σ_t) is (strictly) stationary and consists of non-negative random variables independent of the i.i.d. sequence (Z_t). We refer to [1] for a recent overview of the theory of stochastic volatility models. The

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popular GARCH model has the same structure as in (1.1), but every Z_t feeds into the future volatilities σ_{t+k}, $k \geq 1$, and so (σ_t) and (Z_t) are dependent in this case. However, neither σ_t nor Z_t are directly observable, and therefore it depends on whether one prefers a stochastic volatility, a GARCH or any other model for returns.

In this paper, we consider stochastic volatility models in which σ_t is a strictly stationary regularly varying random sequence. A strictly stationary sequence (X_t) is said to be regularly varying with index $\alpha > 0$ if for every $d \geq 1$, the vector $\mathbf{X}_d = (X_1, \ldots, X_d)'$ is regularly varying with index $\alpha > 0$. This means that there exists a sequence (a_n) with $a_n \to \infty$ and a sequence of non-null Radon measures (μ_d) on the Borel σ-field of $\overline{\mathbb{R}}_0^d = \overline{\mathbb{R}}^d \setminus \{0\}$ such that for every $d \geq 1$,

$$n P(a_n^{-1}\mathbf{X}_d \in \cdot) \xrightarrow{v} \mu_d(\cdot),$$

where $\xrightarrow{v}$ denotes vague convergence and μ_d satisfies the scaling property $\mu(t\cdot) = t^{-\alpha}\mu(\cdot)$, $t > 0$. The latter property justifies the name "regular variation with index $\alpha > 0$". The sequence (a_n) can be chosen such that $nP(|X_1| > a_n) \to 1$. We refer to [9,10] for more details on regular variation and vague convergence of measures. Assume that (X_t) is regularly varying with index $\alpha > 0$ and normalization (a_n) such that $P(|X| > a_n) \sim n^{-1}$, that the mixing condition $\mathcal{A}(a_n)$ (see [6]) is satisfied, and that the anti-clustering condition

$$\lim_{m\to\infty} \limsup_{n\to\infty} P(\max_{m\leq|t|\leq r_n} |X_t| > y\,a_n \mid |X_0| > y\,a_n) = 0, \quad y > 0, \tag{1.2}$$

holds. Here, (r_n) is an integer sequence such that $r_n \to \infty$, $r_n = o(n)$ which appears in the definition of $\mathcal{A}(a_n)$. Then, Davis and Hsing [4] have presented a rather general approach to the extremes of a strictly stationary sequence (X_t). Mikosch and Rezapour [8] showed that the stochastic volatility model in (1.1) inherits the mixing condition $\mathcal{A}(a_n)$ and regularly varying property of (X_t) from the volatility sequence (σ_t). Here, we will show that the anti-clustering condition of (X_t) can be obtained from the anti-clustering property of the volatility sequence (σ_t) under certain conditions. We also introduce a stochastic volatility model whose volatility is exponential AR(2) which is an extension of the exponential AR(1) described in [8]. We will then show that this model is a regularly varying random sequence and satisfies the mixing and anti-clustering conditions described above. Note that we can also define exponential AR(p) in a similar manner, but we cannot obtain a similar compact form (as obtained in Lemma 3.1) for exponential AR(p) with $p > 2$, and so we cannot show that this model also has the regular variation, mixing, and anti-clustering conditions. This remains as an open problem.

2. Regular variation of stochastic volatility model

In this section, we show that the stochastic volatility model in (1.1) inherits the anti-clustering property of (X_t) from the volatility sequence (σ_t).

Theorem 2.1. *Consider the stochastic volatility model in (1.1) and assume that (σ_t) is strictly stationary and regularly varying with index α. Moreover, assume that (σ_t) satisfies the anti-clustering condition in (1.2), $r_n = o(n^C)$ for some positive constant C, and $nE(|Z|^p I_{|Z|>M^{-1}a_n}) = O(1)$ for some positive M and $p < \frac{\alpha}{2}$. Then, (X_t) satisfies the anti-clustering condition.*

Proof. We have

$$P(\max_{m<|t|<r_n} |X_t| > a_n \mid |X_0| > a_n) \sim \frac{P(\max_{m<|t|<r_n} |Z_t|\sigma_t > a_n, |Z_0|\sigma_0 > a_n)}{P(\sigma > a_n)}. \tag{2.1}$$

Let

$$A = \{\max_{m<|t|<r_n} |Z_t|\sigma_t > a_n, |Z_0|\sigma_0 > a_n\},$$

$$B_1 = \{\sigma_0 > Ma_n, \max_{m<|t|<r_n} \sigma_t > Ma_n\},$$

$$B_2 = \{\sigma_0 \leq Ma_n, \max_{m<|t|<r_n} \sigma_t \leq Ma_n\},$$