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On estimating extremal dependence structures by parametric spectral measures

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ABSTRACT

Estimation of extreme value copulas is often required in situations where available data are sparse. Parametric methods may then be the preferred approach. A possible way of defining parametric families that are simple and, at the same time, cover a large variety of multivariate extremal dependence structures is to build models based on spectral measures. This approach is considered here. Parametric families of spectral measures are defined as convex hulls of suitable basis elements, and parameters are estimated by projecting an initial nonparametric estimator on these finite-dimensional spaces. Asymptotic distributions are derived for the estimated parameters and the resulting estimates of the spectral measure and the extreme value copula. Finite sample properties are illustrated by a simulation study.

1. Introduction

Extreme value copulas provide a suitable general approach to modeling multivariate extremes. Various nonparametric methods for estimating extreme value copulas have been proposed in the last few years [18,10,11,3] (also see [16,5,13] for related approaches). In practical applications, such as for instance operational risk or rare natural disasters, one is however often in a situation where available data are sparse. Nonparametric methods generally require a fairly large sample size in order to be reliable. For small samples and in situations where one may have some idea about plausible properties

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of the distribution, parametric methods are likely to yield more accurate results. An approach to parametric inference for extreme value copulas is discussed for instance in [2].

One of the key issues is how to define parametric families that are simple and at the same time general enough to cover a large variety of multivariate dependence structures in the extremes. For instance, some of the most popular models are based on Archimedean copulas, which all correspond to the same type of extremal dependence structure, characterized by the Gumbel copula [7]. One way of achieving more flexibility in the extremes is to build models based on spectral measures. This is the approach taken here. For related work see e.g. [6,12,11].

More specifically, the idea pursued in the following is to select a finite number of suitable spectral measures as basis elements and to use their convex combinations as a parametric family of dependence structures. Given a sufficiently large number of such basis elements, any spectral measure can be approximated by a weighted sum. Estimation of the coefficients can then be carried out by projecting a nonparametric estimator, such as the one in [3], on the finite-dimensional space generated by the basis elements. If the number of basis elements in the model is large (and increasing with the sample size), then projecting the original nonparametric estimator can be considered as a discretization technique. This is the setting in [6,11].

On the other hand, an appropriate model with a small number of basis elements can have the advantage of dimension reduction. Given a reasonable parametric model with a small number of parameters, one can reduce the variability of a nonparametric estimator by projecting it on a low-dimensional space. This is the approach studied here. We define explicit parameter estimators in the low-dimensional setting and study the asymptotic distribution of the resulting estimators of the dependence structure. To illustrate the potential advantage of dimension reduction, we construct an example with three basis elements and compare a nonparametric estimator with its low-dimensional projection in a simulation study.

Note that in principle any nonparametric estimator (cf. [5,3,13,18,10,11]) can be used as a starting point. Depending on the nonparametric method used in the projection, the marginal distributions are either known or estimated from the observed data. The asymptotic results given below only require that a functional limit theorem in a suitable topology holds for the initial estimator.

The paper is organized as follows. Basic definitions and concepts of multivariate extreme value theory are summarized in Section 2. Parametric models in the spectral domain and a corresponding parametric estimator are introduced in Section 3. Asymptotic results, including consistency and a central limit theorem, are derived in Section 4. The theoretical results are illustrated by simulations for a specific model in Section 5. Final remarks in Section 6 with a discussion of some open problems conclude the paper.

2. Basic definitions

Consider a sample $\mathbf{X}_1, \ldots, \mathbf{X}_n$ consisting of iid realizations $\mathbf{X}_i = \left(X_{i,1}, \ldots, X_{i,d}\right)^T$ of a d-dimensional random vector $\mathbf{X} = (X_1, \ldots, X_d)^T \in \mathbb{R}^d$ with marginal distributions $F_1, \ldots, F_d$ and copula $C_{\mathbf{X}}$. That is, $F_j(t) = P\left(X_j \le t\right)$ for $j = 1, \ldots, d$ and $t \in \mathbb{R}$, and

$$P\left(\mathbf{X} \le \mathbf{x}\right) = C_{\mathbf{X}}\left(F_1\left(x_1\right), \ldots, F_d\left(x_d\right)\right)$$

for $\mathbf{x} \in \mathbb{R}^d$. The notation $\mathbf{x} \le \mathbf{y}$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ means $x_j \le y_j$ for $j = 1, \ldots, d$. The transposition operator $(\cdot)^T$ in $\mathbf{X} = (X_1, \ldots, X_d)^T$ indicates that $\mathbf{X}$ is considered as a column vector. Distinguishing columns and rows will be useful in some calculations later on.

The vector $\mathbf{M}_n = \left(M_{n,1}, \ldots, M_{n,d}\right)^T$ of componentwise maxima

$$M_{n,j} = \max_{i=1,2,\ldots,n} X_{i,j}$$

then has marginal distributions $P(M_{n,j} \le t) = F_j^n(t)$ and a copula $C_{\mathbf{M}_n}(u)$ given by

$$P\left(\mathbf{M}_n \le \mathbf{x}\right) = C_{\mathbf{M}_n}\left(F_1^n\left(x_1\right), \ldots, F_d^n\left(x_d\right)\right) = C_{\mathbf{X}}^n\left(F_1\left(x_1\right), \ldots, F_d\left(x_d\right)\right).$$