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Robust parameter change test for Poisson autoregressive models

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ABSTRACT

This study considers the problem of testing for a parameter change in Poisson autoregressive models in the presence of outliers. For this purpose, we propose a cumulative sum test based on the robust estimator introduced by Kang and Lee (2014a), and derive its limiting null distribution. Simulation results demonstrate the robust properties of the proposed test.

1. Introduction

In recent years, integer-valued time series models have been popular in various fields such as finance, insurance, and epidemiology. For example, in epidemiology, these models have been widely used in modeling the monthly number of people with a certain disease. As is addressed in Kang and Lee (2014b), integer-valued time series models can suffer from structural or parameter changes in underlying models due to, for example, changes in the quality of health care and the state of patients' health in epidemiology. Since such changes affect undesirably statistical inference, change point detection has become one of the important task in empirical analysis. For this reason, the problem has attracted much attention from many authors. In the field of integer-valued time series models, Kang and Lee (2009, 2014b) proposed a cumulative sum (CUSUM) test for detecting change points in random coefficient integer-valued autoregressive model and Poisson autoregressive model, respectively. Fokianos and Fried (2010, 2012) investigated the detection of intervention effects in linear and log-linear Poisson autoregressive models, respectively. For a general review of the change point analysis, we refer to Csörgő and Horváth (1997).

This study addresses the problem of detecting a parameter change for Poisson autoregressive models in the presence of outliers. As is widely recognized in the literature, parameter estimation and test procedures are influenced unduly by outliers or extreme values. To overcome the problem, various robust methods have been developed. Concerning change point detection in time series models, Tsay (1988) proposed the procedure for detecting outliers, level shifts, and variance change in a univariate time series, Lee and Park (2001) suggested the CUSUM of squares test based on trimmed observations for the variance change in linear processes. However, to the best of our knowledge, little effort has been made for integer-valued time series models.

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The purpose of this study is to propose a robust test for Poisson autoregressive models. For this, we consider the CUSUM test based on a robust estimator. Kang and Lee (2014a) proposed the minimum density power divergence estimator (MDPDE) for Poisson autoregressive models and demonstrated that the estimator has strong robust properties with little loss in asymptotic efficiency. We use the estimator in constructing the CUSUM test.

This paper is organized as follows. In Section 2, we review the MDPDE for Poisson autoregressive models and its asymptotic properties. In Section 3, we present the MDPDE-based CUSUM test and derive its asymptotic null distribution. In Section 4, we perform a simulation study to compare the performance of the CUSUM tests based on the MDPDE and MLE. Section 5 concludes the paper. The proofs are provided in the Appendix.

2. MDPDE for Poisson autoregressive model

The Poisson autoregressive model is defined by

$$X_t|\mathcal{F}_{t-1} \sim Poisson(\lambda_t), \qquad \lambda_t = f_\theta(\lambda_{t-1}, X_{t-1}) \quad \text{for all } t \in \mathbb{Z}, \tag{1}$$

where f_θ is a known positive function on $[0,\infty) \times \mathbb{N}_0$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, depending on unknown parameter $\theta \in \Theta \subset \mathbb{R}^d$, and $\mathcal{F}_{t-1}$ is the σ-field generated by $\{X_{t-1}, X_{t-2}, \ldots\}$. Model (1) with a linear specification $f(\lambda, x) = w + a\lambda + bx$ is referred to as the INGARCH(1,1) model (cf. Ferland et al., 2006). In what follows, it is assumed that the function f_θ satisfies the following contraction condition, that is,

A1. For all $\theta \in \Theta$, $|f_\theta(\lambda, x) - f_\theta(\lambda', x')| \le \kappa_1|\lambda - \lambda'| + \kappa_2|x - x'|$ for all $\lambda, \lambda' \ge 0$ and $x, x' \in \mathbb{N}_0$, where κ_1 and κ_2 are nonnegative real numbers with $\kappa := \kappa_1 + \kappa_2 < 1$.

Under condition A1, there is a strictly stationary and ergodic solution for model (1) and any order moments of X_t and λ_t are finite (cf. Neumann, 2011; Doukhan et al., 2012).

Denote the true value of θ by θ_0. To estimate the unknown parameter in the presence of outliers, Kang and Lee (2014a) suggested an estimator using the density power divergence proposed by Basu et al. (1998). More specifically, suppose that $X_1, \ldots, X_n$ from (1) are observed. Then, the MDPDE for (1) is given by

$$\hat\theta_{\alpha,n} = \underset{\theta\in\Theta}{\text{argmin}}\, \tilde H_{\alpha,n}(\theta) = \underset{\theta\in\Theta}{\text{argmin}} \sum_{t=1}^n \tilde l_{\alpha,t}(\theta),$$

where

$$\tilde l_{\alpha,t}(\theta) := \begin{cases} \displaystyle\sum_{y=0}^{\infty} \left(\frac{e^{-\tilde\lambda_t}\tilde\lambda_t^y}{y!}\right)^{1+\alpha} - \left(1+\frac{1}{\alpha}\right)\left(\frac{e^{-\tilde\lambda_t}\tilde\lambda_t^{X_t}}{X_t!}\right)^{\alpha}, & \alpha > 0, \\ \tilde\lambda_t - X_t \log\tilde\lambda_t + \log(X_t!), & \alpha = 0, \end{cases} \tag{2}$$

and $\tilde\lambda_t$ are defined recursively by

$$\tilde\lambda_t = f_\theta(\tilde\lambda_{t-1}, X_{t-1}), \quad t \ge 2,$$

with arbitrarily chosen $\tilde\lambda_1$. Hereafter, we use the notations $\tilde\lambda_t(\theta)$ and $\lambda_t(\theta)$ to represent $\tilde\lambda_t$ and λ_t, respectively. The symbol $\|\cdot\|$ denotes the l_2 norm for matrices and vectors, and $E(\cdot)$ is taken under θ_0. Further, we denote by $l_{\alpha,t}(\theta)$ the counterpart of $\tilde l_{\alpha,t}(\theta)$ substituting $\tilde\lambda_t$ with λ_t.

To derive asymptotic results for the MDPDE and the CUSUM statistics below, we make the following assumptions:

A2. $\theta_0 \in \Theta$ and Θ is compact. The function f satisfies

$$f_\theta(\lambda, x) \ge \delta_L > 0$$

for all $\theta \in \Theta$, $\lambda \ge 0$ and $x \in \mathbb{N}_0$.

A3. $E(\sup_{\theta\in\Theta} \lambda_1(\theta)) < \infty$ and $E(\sup_{\theta\in\Theta} \tilde\lambda_1(\theta)) < \infty$.

A4. $\lambda_t(\theta) = \lambda_t(\theta_0)$ *a.s.* implies $\theta = \theta_0$.

A5. θ_0 is an interior point of Θ.

A6. $\lambda_t(\theta)$ is twice continuously differentiable with respect to θ and satisfies

$$E\left(\sup_{\theta\in\Theta}\left\|\frac{\partial\lambda_t(\theta)}{\partial\theta}\right\|\right)^4 < \infty \quad \text{and} \quad E\left(\sup_{\theta\in\Theta}\left\|\frac{\partial^2\lambda_t(\theta)}{\partial\theta\partial\theta^T}\right\|\right)^2 < \infty.$$

A7. There exists an integrable random variable V and a real number ρ with $0 < \rho < 1$, such that, a.s.,

$$\sup_{\theta\in\Theta}\left\|\frac{\partial\lambda_t(\theta)}{\partial\theta} - \frac{\partial\tilde\lambda_t(\theta)}{\partial\theta}\right\| \le V\rho^t \quad \text{and} \quad \sup_{\theta\in\Theta}\left\|\frac{\partial^2\lambda_t(\theta)}{\partial\theta\partial\theta^T} - \frac{\partial^2\tilde\lambda_t(\theta)}{\partial\theta\partial\theta^T}\right\| \le V\rho^t.$$

A8. $\nu^T \frac{\partial\lambda_t(\theta_0)}{\partial\theta} = 0$ *a.s.* implies $\nu = 0$.