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Pointwise and uniform moderate deviations for nonparametric regression function estimator on functional data

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ABSTRACT

In this paper we establish moderate deviations of regression function estimator on functional data. The pointwise moderate deviation principles of Nadaraya–Watson type estimator is considered by exponential equivalence. Depending on the Vapnik–Chervonenkis size of the class, the uniform moderate deviations are also obtained.

1. Introduction

Regression function estimation is an important problem in data analysis and has been proposed in the literature when the discrete time or the continuous time explanatory random variables take their values in a finite dimensional space. We refer to Bosq (1998) for an account of properties and results. Due to the availability of computing resources, functional modeling has received a great interest in the last few years from mathematical, probabilistic, statistical or physicist points of view.

The modelization of functional variable is becoming more and more popular since the publication of the monograph of Ramsay and Silverman (1997) on functional data analysis. However, the first results concerning the nonparametric regression function were obtained by Ferraty and Vieu (2000), in which the almost complete pointwise consistency of kernel regression estimators was studied when the observations are i.i.d. The asymptotic results including the means squared convergence, with rates, as well as the asymptotic normality of kernel estimators of regression function have been obtained by Ferraty et al. (2007); the asymptotic normality of nonparametric regression estimation under strong mixing conditions has been established by Masry (2005), while Laib and Louani (2010, 2011) studied the consistencies of kernel regression estimation for functional stationary ergodic data. For more details of the present state of the art, we refer to the works of Ferraty and Vieu (2004, 2006), Ferraty et al. (2006) and Ezzahrioui and Ould-Said (2008) and so on.

Now we introduce the regression function estimator on functional data. Let $\{(X_i, Y_i), i \geq 1\}$ be a sequence of independent and identically distributed random vectors where Y_i is real-valued and X_i takes values in some semi-metric abstract space

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$(\mathcal{E}, d(\cdot,\cdot))$, which covers the case of semi-normed spaces of possibly infinite dimension (e.g., Hilbert or Banach spaces). Assume that $E(|Y_1|) < \infty$ and that, for a fixed $x \in \mathcal{E}$, the regression function $r(x) = E(Y_1|X_1 = x)$ exists. The Nadaraya–Watson type estimator of $r(x)$ has been introduced by Ferraty and Vieu (2000), based on the usual finite dimensional smoothing ideas. It is defined by

$$r_n(x) = \frac{\sum_{i=1}^{n} Y_i K\left(\frac{d(x,X_i)}{h_n}\right)}{\sum_{i=1}^{n} K\left(\frac{d(x,X_i)}{h_n}\right)} := \frac{\varphi_n(x)}{f_n(x)}$$

where K is a real-valued kernel function, h_n is the bandwidth parameter (which goes to 0 as n goes to infinity), and

$$\varphi_n(x) = \frac{1}{n\phi(h_n)} \sum_{i=1}^{n} Y_i K\left(\frac{d(x,X_i)}{h_n}\right), \qquad f_n(x) = \frac{1}{n\phi(h_n)} \sum_{i=1}^{n} K\left(\frac{d(x,X_i)}{h_n}\right).$$

Here ϕ is a positive function that will be defined below. The pointwise consistency of kernel regression estimator was studied by Ferraty and Vieu (2000), hereafter, it is natural in this setting of functional data to study the rate of convergence. Since uniform consistency results have been successfully used in standard nonparametric setting in order to derive asymptotic properties for data-driven bandwidth choice, additive modeling or multi-step estimation, it is an interesting problem to study the uniform consistency properties of kernel regression estimator. In this paper we will study the asymptotic efficiency of the tail probability of kernel regression estimator using moderate deviations which are often considered as an intermediate result between the central limit theorem and the large deviations principle.

Let us first recall the concept of large and moderate deviations. A sequence $\{Z_n, n \geq 1\}$ of $\mathbb{R}$-valued random variables is said to satisfy a large deviation principles (LDP) with speed v_n and rate function I if for any closed set $F \subset \mathbb{R}$,

$$\limsup_{n\to\infty} v_n^{-1} \log\left(P\Big(Z_n \in F\Big)\right) \leq -\inf_{x\in F} I(x),$$

and any open set $G \subset \mathbb{R}$,

$$\liminf_{n\to\infty} v_n^{-1} \log\left(P\Big(Z_n \in G\Big)\right) \geq -\inf_{x\in G} I(x).$$

Let a_n be a nonrandom sequence that goes to infinity, if there exists function $c(n)$, and $\big(a_n(Z_n - c(n))\big)$ satisfies an LDP, then Z_n is said to satisfy a moderate deviation principles (MDP). Roughly speaking, the MDP for Z_n is the LDP for $\big(a_n(Z_n - c(n))\big)$. The large deviations behavior of the Nadaraya–Watson estimate of the regression function have been studied at first by Louani (1999). In the multidimensional case, Mokkadem et al. (2008) obtained the large deviations and moderate deviations of the classical kernel estimator of regression function. Diallo and Louani (2013) studied the pointwise large and moderate deviation principles for the hazard rate function kernel estimator in the right censorship setting. For regression function estimator on functional data, under suitable conditions, Cherfi (2011) estimated the uniform large deviations of $r_n(x)$ on some compact subset of $\mathcal{E}$, Louani and Ould Maouloud (2011) extended the uniform large deviations for $r_n(x)$ by VC-classes condition. For the uniform large deviations and moderate deviations of classical kernel density estimator, we refer to Gao (2003). In this note, we are interested in the problem of establishing the moderate deviations for Nadaraya–Watson type estimator $r_n(x)$. We consider the pointwise moderate deviation principles for $r_n(x)$, and we also get the uniform moderate deviation results depending on the Vapnik–Chervonenkis size of the class.

2. Assumptions and main theorems

In order to study the moderate deviations for the kernel regression function estimator, we need the following assumptions:

(H1) K is a nonnegative bounded differentiable kernel over its support $[0, 1]$. The derivative K' of K exists on the interval $[0, 1]$, and satisfies the condition $K'(t) < 0, \forall t \in [0, 1]$.

(H2) For $x \in \mathcal{E}$ and a real number y, there exist a nonnegative function f_y and a nonnegative real function ϕ tending to zero, as its argument tends to 0, such that, uniformly in y,

(i) $F_x(u|Y = y) = P\big(d(x, X_1) \leq u|Y = y\big) = \phi(u)f_y(x) + o\big(\phi(u)\big)$, as $u \to 0$.

(ii) There exists a nondecreasing bounded function τ_0 such that, uniformly in $u \in [0, 1]$,

$$\frac{\phi(uh_n)}{\phi(h_n)} = \tau_0(u), \quad \text{as } h_n \to 0,$$

and

$$\int_0^1 |K'(u)|\tau_0(u)du < \infty.$$

Assumptions (H1) and (H2) are considered by Louani and Ould Maouloud (2011). (H1) is very usual in nonparametric estimation literature devoted to functional data context, and (H2) involves the small ball techniques related to the fractal dimension. A number of examples of the function ϕ together with the corresponding decomposition of the probability of