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Effectiveness analysis of a system with a special warranty scheme

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ABSTRACT

This paper analyses a two-dissimilar-unit priority redundant system under the cover of a special warranty scheme. The first unit is a priority unit in operation and fails in three modes, namely minor, marginal and catastrophic. The cold standby unit fails in a single mode. The warranty scheme provides replacement cover for the priority unit for catastrophic failure and repair cover for other failure types. It also provides repair cover for the non-priority unit (standby unit) for failure. A single repair/replacement facility is available to cater to the needs of the system. Using regenerative point technique in Markov renewal process we find different measures of effectiveness and the expected profit incurred in $(0, t]$.

1. Introduction

Two-unit standby redundant systems are discussed widely in the literature. Goel and Gupta [2], Goel and Sharma [4], Goel and Gupta [3] and some other researchers studied the two-unit redundant systems under different sets of conditions. The concept of warranty caught the special attention of industrialists and marketing professionals due to increasing competition. Different warranty schemes were discussed for obtaining a 'fair chunk' of the market by winning the confidence of the consumer. Jack and Murthy [5] discussed a sub-optimal servicing strategy under a one-dimensional warranty scheme for the items sold under the warranty period, Chen and Popova [1] studied different maintenance policies with a two-dimensional warranty, and Jack et al. [6] reviewed the literature on different maintenance strategies and two-dimensional warranty schemes. The sub-optimal servicing strategy suggested by Jack and Murthy [5], and termed strategy 6 by Jack et al. [6], divides warranty

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period in three parts—I, II and III. According to this strategy, if the failure occurs in I or II then the unit is always minimally repaired with the possibility of replacement on failure in III. Servicing strategy 5, discussed by Jack et al. [1], divides the warranty period into three parts — if the failure occurs in the first part or the third part then the unit is always minimally repaired with the possibility of replacement on failure in the second part. The warranty scheme may also be applied in respect of different failure modes which did not get much attention earlier. Here we discuss a warranty scheme used in respect of different failure modes which is a modified version of strategy 6 of Jack et al. [6]. This warranty scheme provides replacement cover for the priority unit for catastrophic failure and repair cover for any other failure. It also provides repair cover for the non-priority unit (standby unit) for failure.

In the present paper, a two-dissimilar-unit priority redundant system covered by a new warranty scheme is discussed. Different measures of system effectiveness such as the mean time to system failure, availability, busy period of the repair/replacement facility, expected numbers of visits to the repair facility and expected profit incurred are derived by using regenerative stochastic process. A graphical study of the mean time to system failure and availability is also carried out for a particular numerical example.

2. Assumptions

- The priority redundant system consists of two dissimilar units. The priority unit is put online initially and gets priority in operation. The non-priority unit is kept as cold standby.
- The failure of the priority unit may be minor, marginal or catastrophic. The standby unit fails in a single mode.
- The warranty scheme adopted here provides replacement cover for catastrophic failure of the priority unit. It also provides repair cover for other failures of the priority unit as well as for the non-priority unit (standby unit).
- The switchover time is instantaneous.
- There is single repair/replacement facility.
- A repaired unit is as good as new.
- Failure time distributions for the units are negative exponential while repair and replacement time distributions are general.

3. Notation

Table 1 shows the possible states of the system.

Table 1

State	Description
S_0	Priority unit initially online and the other unit as cold standby
S_1	Priority unit under repair having minor failure and the standby unit operative
S_2	Priority unit under repair having marginal failure and the standby unit operative
S_3	Priority unit under replacement having catastrophic failure and the standby unit operative
S_4	Repair of the priority unit that suffered minor failure continues and the standby unit waits for repair
S_5	Repair of the priority unit that suffered marginal failure continues and the standby unit waits for repair
S_6	Replacement of the priority unit that suffered catastrophic failure continues and the standby unit waits for repair
S_7	Priority unit operative and repair of other unit started
S_8	Priority unit that suffered minor failure waits for repair and repair of standby unit continued
S_9	Priority unit waits for repair having marginal failure and repair of standby unit continued
S_{10}	Priority unit waits for replacement having catastrophic failure and repair of standby unit continued

Up states: S_0, S_1, S_2, S_3, S_7.
Down states: $S_4, S_5, S_6, S_8, S_9, S_{10}$.
Regenerative states: S_0, S_1, S_2, S_3, S_7.
Non-regenerative states: $S_4, S_5, S_6, S_8, S_9, S_{10}$.