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On the equivalence of the BLUEs under a general linear model and its restricted and stochastically restricted models

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ABSTRACT

Necessary and sufficient conditions are derived for the equalities of the best linear unbiased estimators (BLUEs) of parametric functions under a general linear model and its restricted and stochastically restricted models to hold.

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1. Introduction

Throughout this paper, we will use the symbol $\mathbb{R}^{m\times n}$ to denote the set of all $m \times n$ real matrices. For any $\mathbf{A} \in \mathbb{R}^{m\times n}$, the symbols $\mathbf{A}'$, $r(\mathbf{A})$ and $\mathscr{C}(\mathbf{A})$ denote, respectively, the transpose, the rank and the range (column space) of a real matrix $\mathbf{A}$. The Moore–Penrose inverse of $\mathbf{A}$, denoted by $\mathbf{A}^{+}$, is defined to be the unique solution $\mathbf{G}$ to the four matrix equations

$$\text{(i) } \mathbf{AGA} = \mathbf{A}, \qquad \text{(ii) } \mathbf{GAG} = \mathbf{G}, \qquad \text{(iii) } (\mathbf{AG})' = \mathbf{AG}, \qquad \text{(iv) } (\mathbf{GA})' = \mathbf{GA}.$$

Furthermore, let $\mathbf{P_A}$, $\mathbf{E_A}$ and $\mathbf{F_A}$ stand for the three orthogonal projectors $\mathbf{P_A} = \mathbf{AA}^{+}$, $\mathbf{E_A} = \mathbf{I_m} - \mathbf{AA}^{+}$ and $\mathbf{F_A} = \mathbf{I_n} - \mathbf{A}^{+}\mathbf{A}$.

Consider a general Gauss–Markov model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \text{ or in the triple form } \mathscr{M} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \sigma^2\mathbf{V}_{11}\}, \tag{1.1}$$

where $\mathbf{y} \in \mathbb{R}^{n\times 1}$ is an observable random vector, $\mathbf{X} \in \mathbb{R}^{n\times p}$ is a known model matrix of arbitrary rank, $\boldsymbol{\beta} \in \mathbb{R}^{p\times 1}$ is a vector of unknown parameters, $\boldsymbol{\varepsilon} \in \mathbb{R}^{n\times 1}$ is an unobservable random vector with expectation $E(\boldsymbol{\varepsilon}) = \mathbf{0}$ and covariance matrix $\mathrm{Cov}(\boldsymbol{\varepsilon}) = \sigma^2\mathbf{V}_{11}$, where $\mathbf{V}_{11} \in \mathbb{R}^{n\times n}$ is a known nonnegative definite matrix of arbitrary rank, σ^2 is an unknown positive parameter.

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In the investigation of (1.1), the following two cases usually occur:

(a) An extraneous information is available on the unknown parameter vector $\boldsymbol{\beta}$ in the form of a consistent linear matrix equation

$$\mathbf{A}\boldsymbol{\beta} = \mathbf{b}, \tag{1.2}$$

where $\mathbf{A} \in \mathbb{R}^{m\times p}$ is a known matrix and $\mathbf{b} \in \mathbb{R}^{m\times 1}$ is a known vector. The restrictions may arise from one of the following:

(i) Exact knowledge of a single component $\boldsymbol{\beta}_1$ of $\boldsymbol{\beta}$ in (1.1), for example,

$$\boldsymbol{\beta}_1 = \boldsymbol{\beta}_1^*, \qquad \mathbf{b} = \boldsymbol{\beta}_1^*, \qquad \mathbf{A} = [1, 0, \ldots, 0].$$

(ii) In hypothesis testing on a subvector of $\boldsymbol{\beta} = [\boldsymbol{\beta}_1', \boldsymbol{\beta}_2']'$ in (1.1), such as, $\mathbf{H}_0 : \boldsymbol{\beta}_2 = \mathbf{0}$ with

$$\mathbf{b} = \mathbf{0}, \qquad \mathbf{A} = [\mathbf{0}, \mathbf{I}].$$

(iii) Known knowledge of the ratios between certain coefficients in (1.1), such as, $\boldsymbol{\beta}_1 : \boldsymbol{\beta}_2 = 1 : 2$, which can be reformulated as $\mathbf{b} = 0$, $\mathbf{A} = [2, -1, 0, \ldots, 0]$.

The model (1.1) subject to (1.2) is called a restricted linear model, which can be written in the following form

$$\mathscr{M}_b = \left\{ \begin{bmatrix} \mathbf{y} \\ \mathbf{b} \end{bmatrix}, \begin{bmatrix} \mathbf{X}\boldsymbol{\beta} \\ \mathbf{A}\boldsymbol{\beta} \end{bmatrix}, \sigma^2 \begin{bmatrix} \mathbf{V}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right\}. \tag{1.3}$$

Linear models with restrictions were widely investigated from various aspects, see, e.g., Amemiya (1985), Baksalary and Pordzik (1990), Chipman and Rao (1964), Dent (1980), Ravikumar et al. (2000), and Haupt and Oberhofer (2002).

(b) In many models of practical interest, in addition to the sample information of the (1.1), auxiliary information is available that often may be expressed by a linear stochastic restriction of the type

$$\mathbf{r} = \mathbf{B}\boldsymbol{\beta} + \mathbf{e}, \qquad E(\mathbf{e}) = \mathbf{0}, \qquad \mathrm{Cov}(\mathbf{e}) = \sigma^2\mathbf{V}_{22}, \tag{1.4}$$

where $\mathbf{B} \in \mathbb{R}^{m\times p}$ is a known matrix, $\mathbf{e} \in \mathbb{R}^{m\times 1}$ is a random error vector, $\mathbf{V}_{22} \in \mathbb{R}^{m\times m}$ is a known nonnegative definite matrix, $\mathbf{r} \in \mathbb{R}^{m\times 1}$ is a vector of observed values. The vector $\mathbf{r}$ can be interpreted as a random variable with $E(\mathbf{r}) = \mathbf{B}\boldsymbol{\beta}$; see Rao et al. (2008, p. 252).

This supplementation can be expressed as the partitioned model:

$$\mathscr{M}_r = \left\{ \begin{bmatrix} \mathbf{y} \\ \mathbf{r} \end{bmatrix}, \begin{bmatrix} \mathbf{X}\boldsymbol{\beta} \\ \mathbf{B}\boldsymbol{\beta} \end{bmatrix}, \sigma^2 \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix} \right\}. \tag{1.5}$$

Linear stochastic restrictions can be applied to many fields such as economic relations, industrial structures, production planning, and so on. While, as pointed out by Arashi and Tabatabaey (2008) using stochastic linear restriction, one can accomplish an examination and analysis of one' own thoughts and feelings. In addition, one may also have prior information from past investigations or/and long association with the resembling studies, etc. Many discussions on stochastic restrictions in linear models may be found in the literature, see, e.g., Theil and Goldberger (1961), Theil (1963), Rao et al. (2008), Xu and Yang (2007), and Haslett and Puntanen (2010).

Let $\mathbf{K} \in \mathbb{R}^{l\times p}$ be a known matrix. Then the product $\mathbf{K}\boldsymbol{\beta}$ is called a vector of parametric functions of $\boldsymbol{\beta}$ in (1.1), (1.3) and (1.5). In particular, if $\mathbf{K} = \mathbf{I}_p$ or $\mathbf{K} = \mathbf{X}$, then $\mathbf{K}\boldsymbol{\beta}$ is the parameter vector $\boldsymbol{\beta}$ and the mean vector $\mathbf{X}\boldsymbol{\beta}$, respectively. Throughout this paper, we assume that $\mathbf{A}$ in (1.2) is the same as $\mathbf{B}$ in (1.4) and use the notation $\mathrm{BLUE}_*(\mathbf{K}\boldsymbol{\beta})$ to denote the BLUE of $\mathbf{K}\boldsymbol{\beta}$ in model $*$. In our considerations, the scalar σ^2 has no role and so here we simplify the matters by putting $\sigma^2 = 1$.

Much of the literature in linear models has focused on BLUEs under $\mathscr{M}$ and $\mathscr{M}_b$. For example, Baksalary and Kala (1979) derived the expression of the BLUE of $\mathbf{K}\boldsymbol{\beta}$ under $\mathscr{M}_b$; Mathew (1983) investigated the equality of the BLUEs of $\mathbf{X}\boldsymbol{\beta}$ under $\mathscr{M}$ and $\mathscr{M}_b$ and a necessary and sufficient condition was given; Baksalary and Pordzik (1989) considered the equivalence of the BLUEs of $\mathbf{K}\boldsymbol{\beta}$ under $\mathscr{M}$ and $\mathscr{M}_b$. Recently, Tian (2010) revisited the equivalence of the BLUEs of $\mathbf{K}\boldsymbol{\beta}$ under $\mathscr{M}$ and $\mathscr{M}_b$ and obtained some new equivalent conditions by rank of matrix; Tian (2012) gave some results on equivalence of the BLUEs under the explicitly and implicitly restricted linear models.

Clearly, linear equality restrictions (1.2) are often discomforting in some cases such as economic relations, industrial structures, production planning, and so on. Thus, it is of great importance to investigate linear stochastic restrictions. On some assumptions, some authors compared the superiority of BLUEs of $\boldsymbol{\beta}$ in the sense of the covariance matrix under $\mathscr{M}$ and $\mathscr{M}_r$, as well as under $\mathscr{M}_b$ and $\mathscr{M}_r$, see, e.g., Baksalary (1984), Liski (1989), Trenkler (1993) and Xu and Yang (2007). There is, however, an almost non-existent literature on equality of BLUEs under $\mathscr{M}$ and $\mathscr{M}_r$, as well as under $\mathscr{M}_b$ and $\mathscr{M}_r$. This can be important on statistical inference and its applications. Thus, we deal with them in the paper.

This paper is organized as follows. In Section 2, we give some general expressions of the BLUEs under (1.1), (1.3) and (1.5). In Section 3, we derive necessary and sufficient conditions for the BLUEs to equal under (1.1), (1.3) and (1.5). Section 4 includes the final comments of this paper. The proofs of main results are given in the Appendix.

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