Statistics and Probability Letters 83 (2013) 115–122

Contents lists available at SciVerse ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Robust stabilization with a general decay of mild solutions of stochastic evolution equations

T.E. Govindan [a,*], N.U. Ahmed [b]

[a] Instituto Politécnico Nacional, Mexico City, Mexico
[b] University of Ottawa, Ottawa, Canada

ARTICLE INFO

Article history:
Received 31 May 2012
Received in revised form 22 August 2012
Accepted 22 August 2012
Available online 31 August 2012

MSC:
60H20
34K50
93E15

Keywords:
Stochastic evolution equations in infinite dimensions
Existence and uniqueness of a mild solution
Exponential stability of second moments with a general decay
Robust stabilization

ABSTRACT

In this paper we study the question of robust stabilization of infinite dimensional stochastic systems against uncertainty induced by relatively bounded perturbations of the principal operator determining the system. We present results on state feedback robust stabilization with a general decay. Two examples are included to illustrate the theory.

1. Introduction

Existence, stability and stabilizability problems of mild solutions of stochastic evolution equations (SEEs) in infinite dimensions has been investigated by several authors, see, for example, Ahmed (1991, 1992, 2000), Da Prato and Zabczyk (1992) and Ichikawa (1982), among others. SEEs are well-known to model stochastic processes observed in the study of dynamic systems arising from many areas of science, engineering and finance. The aim of this paper is to study SEEs and consider feedback control problems of unstable systems subject to uncertain perturbations of the unbounded principal operator like A in the following model,

$$dx(t) = [(A+P)x(t) + Bu(t) + f(t, x(t))]dt + g(t, x(t))dw(t), \quad t > 0; \tag{1.1}$$

$$x(0) = x_0, \tag{1.2}$$

where P is contained in a family of unbounded operators $\mathcal{P}$ and u is the control. This is an uncertain stochastic dynamic system. The uncertainty is denoted by the set $\mathcal{P}$ which consists of a family of relatively A-bounded perturbations. This can be interpreted as lack of completeness of the model due to lack of knowledge of the lower order differential operators in a PDE situation. Our main objective here is to construct a state feedback control law for the system (1.1)–(1.2) so that in the presence of uncertain perturbation ($P \in \mathcal{P}$) of the unbounded operators, which may be viewed as structural perturbation,

* Corresponding author.
E-mail address: tegovindan@yahoo.com (T.E. Govindan).

0167-7152/$ – see front matter
doi:10.1016/j.spl.2012.08.019

the system is stable in the second moments with a general decay. In other words, the system is stable uniformly with respect to $P \in \mathcal{P}$ which we call robustness in stability. This result is new and is proved in Theorem 3.6. In order to study this problem, we need to cover some basic existence and regularity properties of solutions of the uncontrolled and unperturbed version of the system (1.1)–(1.2) as described below:

$$dx(t) = [Ax(t) + f(t, x(t))]dt + g(t, x(t))dw(t), \quad t > 0; \tag{1.3}$$

$$x(0) = x_0. \tag{1.4}$$

First we consider the question of existence and uniqueness of mild solutions of the basic system (1.3)–(1.4) and then present sufficient conditions for stability of the moments of the solution with a general decay. To prove the existence and uniqueness of mild solutions, we use the classical Banach fixed point theorem combined with the well known theory of a stochastic convolution integral given by Da Prato and Zabczyk (1992).

The format of the rest of the paper is as follows: In Section 2, we give the preliminaries from Ichikawa (1982) and Ahmed (1991). The main result of the paper is established in Section 3. Section 4 deals with two examples.

2. Preliminaries

Let X, Y be a pair of real separable Hilbert spaces and $\mathcal{L}(Y, X)$ the space of bounded linear operators mapping Y into X. For convenience, we shall use the notations $|\cdot|$ and $(\cdot, \cdot)$ for norms and scalar products for both the Hilbert spaces. We write $\mathcal{L}(X)$ for $\mathcal{L}(X, X)$. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. A map $x : \Omega \to X$ is a random variable if it is strongly measurable. Let $x : \Omega \to X$ be a square integrable random variable, that is $x \in \mathcal{L}_2(\Omega, \mathcal{F}, P; X)$. The covariance operator of the random element x is $\mathrm{Cov}[x] = E[(x - Ex) \circ (x - Ex)]$, where E denotes the expectation and $g \circ h \in L(X)$ for any $g, h \in X$ is defined by $(g \circ h)k = g(h, k)$, $k \in X$. Then $\mathrm{Cov}[x]$ is a selfadjoint nonnegative trace class (or nuclear) operator and $\mathrm{tr}\,\mathrm{Cov}[x] = E|x - Ex|^2$, where tr denotes the trace. The joint covariance of any pair $\{x, y\} \subset \mathcal{L}_2(\Omega, \mathcal{F}, P; X)$, is defined as $\mathrm{Cov}[x, y] = E[(x - Ex) \circ (y - Ey)]$.

Let I be a subinterval of $[0, \infty)$. A stochastic process $\{x\}$ with values in X is a family of random variables $x(t)$, $t \in I$, taking values in X. Let $\mathcal{F}_t$, $t \in I$, be a family of increasing sub σ-algebras of the sigma algebra $\mathcal{F}$. A stochastic process $x(t)$, $t \geq 0$, is adapted to $\mathcal{F}_t$ if $x(t)$ is $\mathcal{F}_t$ measurable for all $t \in I$.

A stochastic process $w(t)$, $t \geq 0$, in a real separable Hilbert space Y is a Wiener process if (a) $w(t) \in \mathcal{L}_2(\Omega, \mathcal{F}, P; Y)$ and $Ew(t) = 0$ for all $t \geq 0$, (b) $\mathrm{Cov}[w(t) - w(s)] = (t - s)W$, $W \in \mathcal{L}_1^+(Y)$ is a nonnegative nuclear operator, (c) $w(t)$ has continuous sample paths, and (d) $w(t)$ has independent increments. The operator W is called the incremental covariance (operator) of the Wiener process $w(t)$. Then w has the representation $w(t) = \sum_{n=1}^{\infty} \beta_n(t) e_n$, where $\{e_n\}$ $(n = 1, 2, 3, \ldots)$ is an orthonormal set of eigenvectors of W, $\beta_n(t)$, $n = 1, 2, 3, \ldots$ are mutually independent real valued Wiener processes with incremental covariance $\lambda_n > 0$, $We_n = \lambda_n e_n$ and $\mathrm{tr}\, W = \sum_{n=1}^{\infty} \lambda_n$.

A semigroup $\{S(t), t \geq 0\}$ is said to be exponentially stable if there exist positive constants $\{M, a\}$ such that $\|S(t)\| \leq Me^{-at}$, $t \geq 0$, where $\|\cdot\|$ denotes the operator norm in $\mathcal{L}(X)$. If $M = 1$, the semigroup is said to be a contraction semigroup.

Now we consider the system (1.3)–(1.4). Let $A : D(A) \subseteq X \to X$ be the infinitesimal generator of a strongly continuous semigroup $\{S(t), t \geq 0\}$ in X. Let the functions f and g with $f : R^+ \times X \to X$, and $g : R^+ \times X \to \mathcal{L}(Y, X)$ be Borel measurable maps.

Next, we introduce the well-known notion of a mild solution for the system (1.3)–(1.4).

Definition 2.1. A stochastic process $x : [0, T] \to X$ defined on the probability space $(\Omega, \mathcal{F}, P)$ is called a mild solution of Eq. (1.3) if

(i) x is jointly measurable and $\mathcal{F}_t$-adapted and its restriction to the interval $[0, T] \equiv I$ satisfies $\int_0^T |x(t)|^2 dt < \infty$, a.s., and
(ii) x satisfies the integral equation

$$x(t) = S(t)x_0 + \int_0^t S(t - s)f(s, x(s))ds + \int_0^t S(t - s)g(s, x(s))dw(s), \quad t \in [0, T] \text{ a.s.}$$

3. Robust stabilization by state feedback

In the preceding section we defined that the principal operator A is the infinitesimal generator of an exponentially stable semigroup. We can avoid this if the system is controllable and stabilizable. We consider the question of stabilizability of second moments with a polynomial decay by use of state feedback without assuming exponential stability of the semigroup generated by the operator A. We refer to Ahmed (1991, 2000) and Ahmed and Skowronski (1994) for some earlier study on exponential stabilizability.

3.1. Uncontrolled system

In order to carry out the program as stated, first we consider the uncontrolled system (1.3)–(1.4). We assume that x_0 is $\mathcal{F}_0$- measurable such that $E|x_0|^p < \infty$ for some integer $p \geq 2$.