Statistics and Probability Letters 82 (2012) 1331–1336

Contents lists available at SciVerse ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

The compound Pascal model with dividends paid under random interest

Xianmin Geng*, Ying Wang

College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

ARTICLE INFO

Article history:
Received 13 November 2011
Received in revised form 22 March 2012
Accepted 27 March 2012
Available online 4 April 2012

MSC:
60K30
90A46

Keywords:
Compound Pascal model
Dividend
Ruin probability
Joint distribution
Lundberg inequality

ABSTRACT

Consider a discrete time risk model under random interest based on the compound Pascal model. The insurer pays a dividend of 1 with a probability q_0 when the surplus is greater than or equal to a non-negative b. In addition, the effect of interest is considered in our model. We derive recursion formulas for the ruin probability, and the joint distribution of the surplus before ruin and the deficit at ruin. Further, we give the generalized Lundberg inequalities for the ruin probability when $q_0 = 1$.

1. Introduction

During the past few decades, the risk model in the presence of dividend payments has become a topic of interest. The pioneer work can be traced to De Finetti (1957). Since then many research studies have been carried out on this topic; see e.g., Gerber (1981), Siegl and Tichy (1999), Albrecher and Kainhofer (2002), Lin et al. (2003), Li and Garrido (2004) and Lin and Pavlova (2006). In addition, Frostig (2005) and Li and Lu (2006) considered the continuous time Markov-modulated risk model with dividends which are paid continuously when the surplus exceeds a constant barrier. Frostig (2005) obtained the expected ruin time and the total dividend payments before ruin occurs in the case where the claim size is exponentially distributed. Li and Lu (2006) derived and solved a system of integro-differential equations for the present value of the total dividend payments prior to ruin, given the initial environment state. Tan and Yang (2006) proposed a discrete time compound binomial risk model with randomized decisions on paying dividends.

Unlike the existing research studies, in this paper we consider the compound Pascal model with dividends paid under random interest. We have derived recursion formulas for the ruin probability, and the joint distribution of the surplus before ruin and the deficit at ruin. Further, we give the generalized Lundberg inequalities for the ruin probability when $q_0 = 1$. For simplicity, we assume that the insurer will pay a dividend of 1 with a probability q_0, $0 \le q_0 < 1$, in each time period if the surplus is greater than or equal to a non-negative integer b at the beginning of the period. And the interest is controlled by a Markov environment.

* Corresponding author.
E-mail address: gengxm@nuaa.edu.cn (X. Geng).

0167-7152/$ – see front matter
doi:10.1016/j.spl.2012.03.037

Let us first introduce the classical discrete time compound Pascal model through

$$U(n) = u + cn - \sum_{k=1}^{N(n)} X_k, \tag{1.1}$$

for $n = 0, 1, 2, \ldots$, here $U(0) = u$ is the initial surplus; c is a positive integer; $\{X_k; k \geq 1\}$ are independent and identically distributed random variables with common purely discrete distribution $p_X(y) = P\{X = y\}, y = 1, 2, 3, \ldots, F_X(y) = P\{X \leq y\}$ and a finite expectation μ_X; $N(n) = \sum_{k=1}^{n} \xi_k, n > 0$, is the number of claims up to time n and $N(0) = 0$, where ξ_n is the claim number in the time period $(n-1, n]$, $n = 1, 2, \ldots$, and $\{\xi_n; n \geq 1\}$ are independent and identically distributed random variables with geometric distribution $P\{\xi_n = k\} = (1-p)p^k, k = 0, 1, 2, \ldots$. So,

$$P\{N(n) = k\} = \binom{n+k-1}{k}(1-p)^n p^k, \quad k = 0, 1, 2, \ldots.$$

And $\{N(n); n \geq 0\}$ is independent of $\{X_k; k \geq 1\}$.

The rest of the paper is organized as follows. In Section 2 we will present some preliminaries of the compound Pascal model with dividends paid under random interest. In Section 3 we will discuss the recursive formulas and the integral equations for the ruin probabilities. In Section 4 we will study the joint distribution of the surplus before ruin and the deficit at ruin. In Section 5 we will give the generalized Lundberg inequality for the infinite time ruin probability when $q_0 = 1$.

2. The model and preliminaries

We assume that all the claims occurring in the time period $(n-1, n]$ occur at time n, and the aggregate claim amount is denoted by $Z_n = \sum_{k=N(n-1)+1}^{N(n)} X_k$. The surplus process $U_b(n)$ is given by

$$U_b(n) = U_b(n-1)(1 + I(A_n)) + c - Z_n - \eta_n \mathbf{1}(U_b(n-1) \geq b), \tag{2.1}$$

where $U_b(0) = u \geq 0$ is the initial surplus, c is the premium rate which is a positive integer, $\mathbf{1}(A)$ is the indicator function of the set A, and η_k $(k \geq 1)$ is a series of randomized decision functions that are mutually independent and identically distributed. $\{\eta_k, k \geq 1\}$ and Z_n are mutually independent. In detail, we denote by $\eta_k = 1$ the event where a dividend of 1 is paid at the time k and denote by $\eta_k = 0$ the event where no dividend is paid at the time k. Assume that $P(\eta_k = 1) = q_0$ and $P(\eta_k = 0) = p_0$, where $0 \leq q_0 < 1$ and $q_0 + p_0 = 1$. $\{A_n, n = 0, 1, 2, \ldots\}$ is a homogeneous Markov chain with the state space $\{a_1, a_2, \ldots, a_M\}$ and the transfer probability p_{st}, where $p_{st} = P\{A_{n+1} = a_t | A_n = a_s\}, n = 0, 1, 2, \ldots, s, t = 1, 2, \ldots, M$, and $I(A_n)$ denotes the interest rate in the time period $(n-1, n]$. Further, $\{A_n\}$ and $\{Z_n\}$ are assumed to be independent, and $\{N(n); n \geq 0\}$ is independent of $\{X_k; k \geq 1\}$.

It is reasonable that the randomized decision functions η_k $(k \geq 1)$ are brought in for deciding on the periods with a dividend. The return on the investment for an insured in each time period can be regarded as being stochastic. If the return on the investment in the present period is greater than or equal to a given level, then the insurer will pay a benefit of 1 to the insured, and if the surplus is smaller than b at the beginning of the present period, then a decision for paying a benefit of 1 will be canceled. Certainly, the randomized decisions can also be related to some other things that occur with probability q_0 and do not occur with probability p_0 in each time period, for example natural disasters. It is worth noting that if $b = \infty$ or $q_0 = 0$, the risk model (2.1) is reduced to the following risk model with a Markov chain interest:

$$U(n) = u \prod_{j=1}^{n}(1 + I(A_j)) + \sum_{j=1}^{n}(c - Z_j) \prod_{m=j+1}^{n}(1 + I(A_m)).$$

The general discrete time model with a Markov chain interest model has been studied by Cai and Dickson (2004).

Lemma 1. *$\{Z_n, n = 1, 2, \ldots\}$ is a sequence of independent and identically distributed random variables with the common distribution density*

$$w(z) = P\{Z_n = z\} = \begin{cases} 1-p & z = 0 \\ \sum_{j=1}^{\infty} p_X^{*(j)}(z)(1-p)p^j & z = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

$$W(z) = P\{Z_n \leq z\} = (1-p) + \sum_{j=1}^{\infty} F_X^{*(j)}(z)(1-p)p^j \quad (z \geq 0), \tag{2.3}$$

where $p_X^{*(j)}(y) = P\{\sum_{k=1}^{j} X_k = y\}, y = 1, 2, 3, \ldots$, and $F_X^{*(j)}(y) = P\{\sum_{k=1}^{j} X_k \leq y\}, y \geq 0$.

Proof. Lemma 1 can easily be obtained from the definition of $\{Z_n\}$. $\square$