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The Hausman–Taylor panel data model with serial correlation

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ABSTRACT

This paper modifies the Hausman and Taylor (1981) panel data estimator to allow for serial correlation in the remainder disturbances. It demonstrates the gains in efficiency of this estimator versus the standard panel data estimators that ignore serial correlation using Monte Carlo experiments.

1. Introduction

The random effects (RE) panel data model assumes that all the explanatory variables are uncorrelated with the random individual effects, while the fixed effects (FE) panel data model assumes that all the explanatory variables are correlated with the random individual effects. Instead of this "all" or "nothing" assumption, the Hausman and Taylor (1981) panel data estimator allows *some* of the explanatory variables to be correlated with the individual effects. One of the main disadvantages of the fixed effects estimator is that it wipes out the effects of time-invariant variables. In contrast, the Hausman and Taylor estimator recaptures the estimates of these time-invariant variables which are important in empirical applications; see Cornwell and Rupert (1988), Egger and Pfaffermayr (2004) and Serlenga and Shin (2007)for three applications of this estimator. This paper extends the Hausman and Taylor (HT) estimator to allow for serial correlation in the remainder disturbances of the AR(1) type. The standard fixed effects (FE) and random effects (RE) panel data models with serial correlation in the remainder disturbances have been considered by Bhargava et al. (1982) and Baltagi and Li (1991), amongst others, see Baltagi (2008). While the fixed effects AR(1) estimator (FE-AR(1)) considered by Bhargava et al. (1982) is consistent for the HT model, it does not provide an estimator of the time-invariant variable coefficients which are usually of interest in most economic applications. The Baltagi and Li (1991) random effects AR(1) (RE-AR(1)) estimator provides estimates of the time-invariant variable coefficients, but these will be consistent only if the individual effects are uncorrelated with all the regressors. The modified HT estimator allowing for AR(1) disturbances (denoted by HT-AR(1)) is more efficient than the HT estimator that ignores this serial correlation. Unlike the FE-AR(1) estimator, it captures the effects of time-invariant variables, and, unlike the Baltagi and Li (1991) RE-AR(1) estimator, it allows for possible correlation between the regressors and the individual effects. In this paper, we report Monte Carlo experiments that demonstrate the gains in efficiency of this HT-AR(1) estimator over the standard HT estimator in the presence of serial correlation.

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2. The model and assumptions

Consider the following Hausman and Taylor (1981) panel data model:

$$y_{it} = X_{it}'\beta + Z_i'\gamma + u_{it}, \quad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{1}$$

where $u_{it} = \mu_i + \nu_{it}$, and β and γ are unknown vectors. The X_{it} are time-varying regressors, while the Z_i are time invariant. This HT model is allowed to have first-order serial correlation in $\{\nu_{it}\}$ of the AR(1) type:

$$\nu_{it} = \rho\nu_{it-1} + \varepsilon_{it}, \qquad |\rho| < 1, \tag{2}$$

where ε_{it} is a white noise process with variance σ_ε^2. The μ_i are independent of the ν_{it} for all i and t. In vector form, Eq. (1) can be written as

$$y_i = W_i\delta + u_i, \quad i = 1, \ldots, N, \tag{3}$$

with

$$u_i = \mu_i \boldsymbol{\iota}_T + \nu_i,$$

where $y_i = (y_{i1}, \ldots, y_{iT})'$, $W_i = \left(X_i, \boldsymbol{\iota}_T Z_i'\right)$, $X_i = (X_{i1}, \ldots, X_{iT})'$, $\delta' = \left(\beta', \gamma'\right)$, $u_i = (u_{i1}, \ldots, u_{iT})'$, and $\boldsymbol{\iota}_T$ is a vector of ones of dimension T. Eq. (3) can be also written as

$$y = W\delta + u, \tag{4}$$

with

$$u = (I_N \otimes \boldsymbol{\iota}_T)\,\mu + \upsilon,$$

where $y = \left(y_1', \ldots, y_N'\right)'$, $W = (X, Z)$, with $X = \left(X_1', \ldots, X_N'\right)'$, $Z = \left(Z_1', \ldots, Z_N'\right)' \otimes \boldsymbol{\iota}_T$, $u = \left(u_1', \ldots, u_N'\right)'$, $\mu = (\mu_1, \ldots, \mu_N)'$, and $\upsilon = (\upsilon_1, \ldots, \upsilon_N)'$.

3. The generalized least squares (GLS) estimator

Assuming that the individual effects μ_i are random with $\mu_i \sim$ i.i.d.$(0, \sigma_\mu^2)$ and $E\left(\mu_i | X_{it}', Z_i'\right) = 0$ for all i and t, the resulting RE GLS-type estimator correcting for AR(1) remainder disturbances will be the best linear unbiased estimator (BLUE); see Baltagi and Li (1991). In fact, this estimator applies the Prais–Winsten (PW) transformation in the first step to transform the remainder AR(1) disturbances into serially uncorrelated classical errors. More specifically, one premultiplies Eq. (4) by

$$C = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & -\rho & 1 & 0 \\ 0 & 0 & 0 & 0 & -\rho & 1 \end{bmatrix}$$

to get

$$y^* = W^*\delta + u^*, \tag{5}$$

where $y^* = (I_N \otimes C)\,y$, $W^* = (I_N \otimes C)\,W$, and $u^* = (I_N \otimes C)\,u$. Using the fact that $C\boldsymbol{\iota}_T = (1-\rho)\,\boldsymbol{\iota}_T^\alpha$, where $\boldsymbol{\iota}_T^{\alpha'} = \left(\alpha, \boldsymbol{\iota}_{T-1}'\right)$ and $\alpha = \sqrt{(1+\rho)\,/\,(1-\rho)}$, the transformed regression disturbances are given by

$$u^* = (I_N \otimes C)\,u = (I_N \otimes C\boldsymbol{\iota}_T)\,\mu + (I_N \otimes C)\,\upsilon = (1-\rho)\left(I_N \otimes \boldsymbol{\iota}_T^\alpha\right)\mu + \upsilon^*, \tag{6}$$

where $\upsilon^* = (I_N \otimes C)\,\upsilon$. As shown in Baltagi and Li (1991), the variance–covariance matrix of the transformed disturbances is given by

$$\Omega^* = E\left(u^*u^{*\prime}\right) = \sigma_\mu^2\,(1-\rho)^2\left(I_N \otimes \boldsymbol{\iota}_T^\alpha \boldsymbol{\iota}_T^{\alpha\prime}\right) + \sigma_\varepsilon^2\,(I_N \otimes I_T)\,, \tag{7}$$

and

$$\sigma_\varepsilon \Omega^{*-1/2} = (I_N \otimes I_T) - \theta_\alpha\left(I_N \otimes \bar{J}_T^\alpha\right),$$

where $E_T^\alpha = I_T - \bar{J}_T^\alpha$, $\bar{J}_T^\alpha = \boldsymbol{\iota}_T^\alpha \boldsymbol{\iota}_T^{\alpha\prime}/d^2$, $d^2 = \alpha^2 + T - 1$, $\theta_\alpha = 1 - \frac{\sigma_\varepsilon}{\sigma_\alpha}$, and $\sigma_\alpha^2 = \sigma_\varepsilon^2 + d^2\,(1-\rho)^2\,\sigma_\mu^2$. Premultiplying the PW transformed observations by $\sigma_\varepsilon \Omega^{*-1/2}$, one gets

$$\sigma_\varepsilon \Omega^{*-1/2} y^* = \sigma_\varepsilon \Omega^{*-1/2} W^*\delta + \sigma_\varepsilon \Omega^{*-1/2} u^*. \tag{8}$$