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# A simple test of optimal hedging policy

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## ABSTRACT

This paper investigates the equivalence between the optimal hedge ratio derived in a risk-return simplification and the optimal hedge ratio using mean–variance analysis. In accordance with this relationship, we develop a simple regression-based test for evaluating the hedging effectiveness of the risk-return hedging. As a result, a $t$-test and an $F$-test are designed to examine the hedge ratio and hedging effectiveness, respectively. An example of hedging is also provided to illustrate this process.



## 1. Introduction

Futures markets supply vital hedge instruments that investors require to make "a reasoned choice in conduct of risk reduction". One of the major research interests in futures markets is to solve the optimal hedging policy for a position to be hedged with futures. Two general methods of determining the optimal hedge ratio are the minimum-variance model and the risk-return approach. The fundamental difference between the risk-return approach and the minimum-variance model is that the risk-return approach highlights the trade-off between return and risk compared to the minimum-variance model, which focuses solely on risk reduction and requires fewer estimates (variances and covariance).

Traditionally, the determination of the optimal hedge ratio of a hedged portfolio has been considered as a solution to a risk reduction problem since Ederington's (1979) proposal of minimum-variance hedging. Researchers have presented ample statistical inferences on this issue based on minimizing portfolio variance (Stulz, 1984; Mcnew and Fackler, 1994; Jong et al., 1997; Brooks et al., 2002; Harris and Shen, 2003; Terry, 2005). Starting with Howard and D'Antonio's (1984) proposal of the relative risk-return measure to determine the optimal hedge ratio, both practitioners and researchers have advanced a variety of risk-return measures to evaluate hedging effectiveness (Chang and Shanker, 1987; Kuo and Chen, 1995; Satyanarayan, 1998). In particular, Kuo and Chen adapt and simplify the Howard–D'Antonio approach for practical settings. However, these studies do not suggest an analytical test-statistic to evaluate hedging effectiveness.

Two main results are obtained in this paper. First, we link the relationship between the revised Howard–D'Antonio model derived by Kuo–Chen and mean–variance analysis according to Heifner (1972) and Kahl (1983). Second, we integrate the mean–variance approach and the Britten-Jones regression approach (1999) into a simple test that is used to estimate the optimal hedge ratio. As a consequence, a $t$-test and an $F$-test are analytically designed to test the hedge ratio and the hedging effectiveness, respectively.

This article is organized as follows. In Section 2, we briefly review Howard–D'Antonio risk-return optimization as well as the relative measure of hedging effectiveness, the Kuo–Chen risk-return simplification, and the Heifner–Kahl mean–variance

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framework. According to this relative measure, we investigate the equivalence between the optimal hedge ratio derived in the Kuo–Chen framework and the optimal hedge ratio using mean–variance analysis. In Section 3, we employ Britten-Jones' regression approach to develop a statistical test that is used to evaluate the Howard–D'Antonio hedging effectiveness. In Section 4, we give an example of how to construct a hedged portfolio using stock index futures. Section 5 concludes.

## 2. Basic features of optimal hedge ratio

### 2.1. Howard–D'Antonio risk-return measure

A riskless asset with a rate of return $i$ is assumed to be available in each period. Howard and D'Antonio consider an investor who wants to maximize the Sharpe ratio problem of a spot position hedged with a futures position. The Sharpe ratio captures both risk and return in a single measure, which is defined as the ratio of a portfolio's excess return per unit of risk associated with the excess return (see Sharpe, 1994). A higher Sharpe ratio of the hedged portfolio indicates higher profitability.

A simplification is suggested by Kuo and Chen in response to criticism of Howard–D'Antonio model regarding the difficulty of estimating futures excess return. Our work is related to the revised Howard–D'Antonio model. Given a portfolio comprising *one* spot position hedged with a futures position $b$, the Kuo–Chen simplification may be summarized as follows:

$$\max_b \frac{\mu_p}{\sigma_p} = \max_b \frac{\mu_s + b\mu_a}{\sqrt{\sigma_s^2 + 2b\sigma_{sa} + b^2\sigma_a^2}} \tag{1}$$

where $\mu_p = \mu_s + b\mu_a$ and $\sigma_p = \sqrt{\sigma_s^2 + 2b\sigma_{sa} + b^2\sigma_a^2}$ are the expected excess return and the risk of a hedged portfolio, respectively. In addition, we employ the following notation for further derivations.

$P_t^s =$ the spot price at period $t$,

$P_t^f =$ the futures price at period $t$,

$\mu_s =$ the expected excess return of spot, $E\left(\frac{P_{t+1}^s - P_t^s}{P_t^s} - i\right)$,

$\mu_a =$ the expected excess return of futures, $E\left(\frac{P_{t+1}^f - P_t^f}{P_t^s}\right)$,

$\mu =$ the vector of expected excess returns, $[\mu_a\ \mu_s]'$, and

$\Sigma =$ the covariance matrix, $\begin{bmatrix} \sigma_a^2 & \sigma_{sa} \\ \sigma_{sa} & \sigma_s^2 \end{bmatrix}$,

$\rho_{sa} =$ the coefficient of correlation, $\frac{\sigma_{sa}}{\sigma_s\sigma_a}$.

Note that there are different ways suggested to calculate the return of futures positions. In this paper, we compute the futures excess return using the Kuo–Chen definition, which is based on the initial margin. Because the futures position not only aligns with the spot position but also does not have to tie cash up with a margin requirement, it appears reasonable for us to apply the spot price $P_t^s$ as a basis to calculate the futures return. Therefore, we consider $\mu_a$ as a proxy for the "expected futures excess return".

**Remark 1.** Some available results related to the solution of Kuo–Chen optimization (1) are listed as follows:

1. The optimal hedge ratio of optimization (1) is

$$b^* = \frac{\sigma_s(\Gamma - \rho_{sa})}{\sigma_a(1 - \Gamma\rho_{sa})}, \tag{2}$$

where

$$\Gamma = \frac{\theta_a}{\theta_s} = \frac{\mu_a}{\sigma_a} \Big/ \frac{\mu_s}{\sigma_s}. \tag{3}$$

We see that $\theta_s = \frac{\mu_s}{\sigma_s}$ and $\theta_a = \frac{\mu_a}{\sigma_a}$ are the Sharpe ratios of spot and futures, respectively. Moreover, $\Gamma$ can be expressed as

$$\Gamma = \frac{\text{Sharpe ratio of futures expected excess return}}{\text{Sharpe ratio of spot expected excess return}}.$$

Note that $\Gamma$ represents a hedging multiplier of the Sharpe ratio of futures against the Sharpe ratio of the spot position. In fact, $\Gamma$ is an intuitive relative because the futures position provides less (more) return for a unit of risk than the spot position if $\Gamma < 1$ $(\lambda > 1)$. Of course, if $\Gamma = 1$, the spot position and futures position are equally attractive.