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Lower and upper bounds of large deviation for sums of subexponential claims in a multi-risk model

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ABSTRACT

In view of the actual condition of the insurance company, a multi-risk model is proposed. The lower and upper bounds for the sums of subexponential claims in this model are given. The proof method is based on the results of the total claim amount under subexponential class.

1. Introduction

The large deviation results are applied to certain problems in insurance and finance which are related to large claims. In classical large deviation, randomly indexed sums(random sums) are denoted by

$$S(t) = \sum_{i=1}^{N(t)} X_i, \tag{1.1}$$

where the claim sizes $X_1, X_2, \dots$ form a sequence of independent identically distributed (i.i.d.) non-negative random variables (rv) with a common distribution function (d.f.) $F(x) = P(X_1 \le x)$ and a finite mean $a = EX_1$, independent of a process of non-negative, integer-valued rv's $\{N(t), t \ge 0\}$. We assume that $\lambda(t) = EN(t) < \infty$, $a(t) = ES(t) = a\lambda(t) < \infty$ for all $t \ge 0$ and $\lambda(t) \to \infty$ as $t \to \infty$. We denote $\sum_{i=1}^{0} X_i = 0$, $S_n = \sum_{1\le i\le n} X_i$, $\overline{F} = 1 - F$ as usual. All rv's are supposed not to degenerate to zero.

Klüppelberg and Mikosch (1997) and Ng et al. (2004) investigated the probabilities of large deviations of $S(t)$ defined by (1.1) and obtained

$$P(S(t) > x + a\lambda(t)) \sim \lambda(t)\overline{F}(x), \quad t \to \infty \tag{1.2}$$

uniformly for $x \ge \gamma\lambda(t)$ for every $\gamma > 0$ under some additional assumptions for $F \in ERV(-\alpha, -\beta)$ and $F \in \mathcal{C}$ respectively.

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In addition, the initial capital of an insurance company $x \to \infty$ is more natural for studying the asymptotics of the finite time ruin probabilities. Thus, Baltrūnas et al. (2008) provided the following model and obtained that for every positive $\mu > 0$,

$$P(S(t) > x + (\mu + a)\lambda t) \sim \lambda t\overline{F}(x + \mu\lambda t), \quad x \to \infty \tag{1.3}$$

uniformly for $t \in [f(x), \gamma x/Q(x)]$ under another four additional assumptions, where $N(t)$ is a renewal counting process and F is the subexponential distribution function, $f(x)$ is an arbitrary infinitely increasing function, and $Q(x) = -\log\overline{F}(x)$. Note that, in general, the zones of uniform convergence in (1.2) and (1.3) are different.

It is these results that motivate our study. All the researchers above always concentrated on the fact that the company provides only one kind of insurance contract. In reality this assumption is not correct, so the large deviation problem of multi-risk model is more valuable. In this paper, we assume that the company has n types of insurance contracts. The ith related loss amounts are denoted by $\{X_{ij}, j \geq 1\}$, which are i.i.d. nonnegative random variables with common subexponential distribution function $F_i(x)$ that has positive finite expectation $a_i, i = 1, \ldots, n$. Let $\{N_i(t), i = 1, \ldots, n\}$ denote renewal counting processes. We assume that $\{X_{ij}, j \geq 1\}_{i=1}^{n}$ and $\{N_i(t), i = 1, \ldots, n\}$ are mutually independent and that $EN_i(t) = \lambda_i(t) \sim \lambda_i t$ as $t \to \infty, i = 1, \ldots, n$.

Returning to our problem in this paper, we consider the following survival probability:

$$P\left(\sum_{1\leq i\leq n}\sum_{1\leq j\leq N_i(t)} X_{ij} > x + \sum_{1\leq i\leq n}(\mu_i + a_i)\lambda_i t\right), \tag{1.4}$$

where $\mu_i > 0$ for $i = 1, \ldots, n$.

Up to now, to the best of our knowledge, only Wang and Wang (2007) considered the large deviation for random sums with different kinds of distributions. They extended the results of Ng et al. (2004) to a multi-risk model. However, little is known about the large deviations for such random sums under subexponential class as $x \to \infty$. Thus, from this point of view, our results extend those of Baltrūnas et al. (2008) to a multi-risk model.

The rest of the paper is organized as follows. In Section 2, we present some related notations and useful results appeared in Baltrūnas et al. (2008) for the subexponential class. The main result and application are given in Section 3. Finally, the proof of the main result is presented in Section 4.

2. Preliminaries

We say X (or its distribution F) is heavy tailed if it has no exponential moments. An important heavy-tailed subclass is the subexponential class $\mathcal{S}$. A distribution function F with support on $[0, \infty)$ belongs to $\mathcal{S}$, if for every $n \geq 2$ (equivalently, for some $n \geq 2$)

$$\lim_{x\to\infty} \frac{P(X_1 + \cdots + X_n > x)}{P(\max(X_1, \ldots, X_n) > x)} = 1.$$

In Baltrūnas et al. (2008), $Q(u) = -\log\overline{F}(u), u \in R_+$ denotes the hazard function of distribution F. They also assumed that there exists a nonnegative function $q : R_+ \to R_+$ such that $Q(u) = \int_0^u q(v)\mathrm{d}v, u \in R_+$. The function q is called the hazard rate of d.f. F. Denote by

$$r := \limsup_{u\to\infty} uq(u)/Q(u) \tag{2.1}$$

a hazard ratio index.

For giving the main result of this paper, we need the following assumptions which generalize the ones in Baltrūnas et al. (2008). For $i = 1, \ldots, n$:

Assumption H_1**.** The claim sizes $\{X_{ij}, j \geq 1\}$ form a sequence of independent identically distributed nonnegative random variables with a common distribution function $F_i(u) = P(X_{ij} \leq u)$, which has a finite mean $a_i = EX_{i1}$ and a finite second moment $EX_{i1}^2 < \infty$.

Assumption H_2**.** The inter-occurrence times $\theta_{i1} = T_{i1}, \theta_{i2} = T_{i2} - T_{i1}, \theta_{i3} = T_{i3} - T_{i2}, \ldots$ are i.i.d. nonnegative random variables with mean $1/\lambda_i$ and finite second moment $E\theta_{i1}^2 < \infty$. In addition, $\{\theta_{ij}, j \geq 1\}$ are mutually independent of $\{X_{ij}, j \geq 1\}$. The random variables $\left\{T_{ik} = \sum_{j=1}^{k}\theta_{ij}, k \geq 1\right\}$ constitute a renewal counting process $N_i(t) = \sharp\{k = 1, 2, \ldots : T_{ik} \in (0, t]\}, t \geq 0$ with a mean function $\lambda_i(t) = EN_i(t)$, for which $\lambda_i(t) \sim \lambda_i t$ as $t \to \infty$.

Assumption A. The distribution F_i is subexponential and satisfies the following conditions:

$$r_i < 1/2; \quad \liminf_{u\to\infty} uq_i(u) \geq \begin{cases} 2 & \text{if } r_i = 0, \\ 4/(1 - r_i) & \text{if } r_i \neq 0. \end{cases}$$