Available online at www.sciencedirect.com
SCIENCE DIRECT®

Statistical Methodology 3 (2006) 174–185

Statistical Methodology
Official Journal of the International Indian Statistical Association

www.elsevier.com/locate/stamet

A two phases queueing system with Bernoulli vacation schedule under multiple vacation policy[☆]

Gautam Choudhury*, Madhuchanda Paul

Mathematical Sciences Division, Institute of Advanced Study in Science and Technology, Paschim Boragoan, Guwahati – 781035, Assam, India

Received 30 December 2003; received in revised form 6 June 2005; accepted 16 September 2005

Abstract

This paper deals with a single server Poisson arrival queue with two phases of heterogeneous service along with a Bernoulli schedule vacation model, where after two successive phases service the server either goes for a vacation with probability p $(0 \leq p \leq 1)$ or may continue to serve the next unit, if any, with probability $q(= 1 - p)$. Further the concept of multiple vacation policy is also introduced here. We obtained the queue size distributions at a departure epoch and at a random epoch, *Laplace Stieltjes Transform* of the waiting time distribution and busy period distribution along with some mean performance measures. Finally we discuss some statistical inference related issues.

MSC: primary 60K25; secondary 90B22, 68M20

Keywords: $M/G/1$ queue; Heterogeneous service; First phase of service; Second phase of service; Bernoulli schedule vacation; Multiple vacation policy; Statistical analysis

1. Introduction

Recently Madan [15,16] studied the $M/G/1$ queueing system with two phases of heterogeneous service viz. first phase of service (*FPS*) followed by a second phase of service (*SPS*) under a Bernoulli vacation schedule. In fact the Bernoulli vacation model was first studied by Keilson and Servi [10], where they introduced the concept of modified service time.

☆ This work is supported by Department of Atomic Energy, Govt of India, NBHM. Project No. 48/2/2001/R&DII/1211.
* Corresponding author. Tel.: +91 031 2513123; fax: +91 0361 2222660.
E-mail addresses: choudhuryg@yahoo.com (G. Choudhury), to_madhu_paul@yahoo.com (M. Paul).

1572-3127/$ - see front matter
doi:10.1016/j.stamet.2005.09.003

Considerable efforts have been devoted to study this model by Servi [20], Ramaswami and Servi [18], Takagi [21] and Doshi [5] among others.

Another vacation model which is a subset of this Bernoulli vacation model is the multiple vacation model, where the server keeps on taking vacations until the server finds no units after returning from a vacation. This type of model has been studied extensively in the past, since Levy and Yechaili [13]. One of the most remarkable results that concerned these types of models is the "*stochastic decomposition result*". Fuhrmann and Cooper [7] were first to establish this result for $M/G/1$ type queue with generalized vacations.

Recently, there have been several contributions considering queueing systems of $M/G/1$ type in which the server may provide a *SPS*. A number of researchers including Madan [14], Medhi [17], Choudhury [3], Krishnakumar and Arivudainambi [12], Krishna and Lee [11], Selvam and Sivasankaran [19], Doshi [6] and many others studied this type of model under different situations. The motivation for this type of model comes from some computers and networks where messages are processed in two stages (phases) by a single server.

Although some aspects of the $M/G/1$ queueing system with two phases of service under Bernoulli vacation schedule have been studied by Madan [15,16], still some questions need to be addressed. Thus in this paper we propose to study such a single server two phases of heterogeneous service queue with Bernoulli vacation schedule, where the concept of multiple vacation policy is also introduced. Unlike Madan [15,16], ours is completely different from him, where after two successive phases of service or after first vacation, the server may go for further vacations till it finds at least one unit in the system. Using Kendall's notation, the model considered in this paper is a $M/(G_1, G_2)/V_M/1(BS)$ queue, where *BS* represents Bernoulli schedule vacation and V_M represents vacation time under multiple vacation policy.

Further, from the practical point of view the type of model can be utilized as a model building of a production process. For example, consider a production process, where the machine producing certain items may require two phases of service such as preliminary checking (*FPS*) followed by the usual process (*SPS*) to complete the processing of the raw materials. It may so happen that the process either needs to be stopped for overhauling and maintenance of the system after these two phases of service or may continue the further processing of the raw materials if there is no fault in the system. This overhauling can be utilized as a vacation time in our system. There may be many other situations such as digital communication or a data transmission system which involves two phases of service. Further, Ghafir and Silio [8] recognized its application in Multiple Access Ring Networks.

The rest of the paper is organized as follows. In Section 2 we give a brief description of the mathematical model. Section 3 deals with the queue size distribution at a departure epoch. The stationary queue size distribution at a random epoch is derived in Section 4. We derive the Laplace–Stieltjes transform (*LST*) of the busy period distribution and waiting time distribution in Section 5. Finally, some statistical inference related issues of this model are discussed in Section 6.

2. The mathematical model

We consider an $M/G/1$ queueing system, where arrival occurs according to a Poisson process with arrival rate 'λ' and the server provides to each unit two phases of heterogeneous service in succession, the first phase service (*FPS*) followed by the second phase service (*SPS*). The service discipline is assumed to be *FCFS*. Further, it is assumed that the service time S_i $(i = 1, 2)$ of the ith phase of service follows a general probability law with distribution function $(d.f)$ $S_i(x)$, *LST*