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Berry–Esseen bound for parameter estimation in some time inhomogeneous diffusions and applications ☆

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ABSTRACT

In this paper, we study the Berry–Esseen bound of the distribution of the maximum likelihood estimation in some inhomogeneous diffusions. We prove that it converges to the normal distribution with an error rate $O((-\log \int_t^T \sigma^2(s)\mathrm{d}s)^{-\frac{1}{2}})$. Moreover, the obtained Berry–Esseen bound can be applied to study the precise asymptotics in the law of iterated logarithm for the maximum likelihood estimation.

1. Introduction

We consider the following time inhomogeneous diffusion process

$$\mathrm{d}X_t^{(\alpha)} = \alpha b(t) X_t^{(\alpha)}\mathrm{d}t + \sigma(t)\mathrm{d}W_t, \qquad X_0^{(\alpha)} = 0, \quad t \in [0, T), \tag{1.1}$$

where W is a standard Brownian motion and the parameter $\alpha \in \mathbb{R}$ is unknown, $b : [0, T) \to \mathbb{R} \setminus \{0\}$ and $\sigma : [0, T) \to (0, +\infty)$ are continuously differentiable functions. We denote by P_α the probability distribution of the solution of (1.1) on $C([0, T), \mathbb{R})$. It is well known that the Hull–White model (1.1) has many applications in practical problems; see, e.g., the book of Bishwal (2007) and the paper of Mishra and Prakasa Rao (1985b).

In Barczy and Pap (2010, 2011), it has been obtained that the maximum likelihood estimator of α based on the observation of $\{X_s, 0 \le s \le t\}$ under P_α is given by

$$\hat{\alpha}_t = \frac{\int_0^t \frac{b(s)}{\sigma^2(s)} X_s^{(\alpha)} \mathrm{d}X_s^{(\alpha)}}{\int_0^t \frac{b^2(s)}{\sigma^2(s)} \left(X_s^{(\alpha)}\right)^2 \mathrm{d}s} = \alpha + \frac{\int_0^t \frac{b(s)}{\sigma(s)} X_s^{(\alpha)} \mathrm{d}W_s}{\int_0^t \frac{b^2(s)}{\sigma^2(s)} (X_s^{(\alpha)})^2 \mathrm{d}s}, \quad t \in (0, T). \tag{1.2}$$

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For the classical Ornstein–Uhlenbeck process, by the squeezing technique due to Pfanzagl (1971) and Bishwal (2000) the Berry–Esseen bound of the maximum likelihood estimator is $O(T^{-\frac{1}{2}})$. For further references, one can see Bose (1986) and Mishra and Prakasa Rao (1985a).

For the time inhomogeneous diffusion process (1.1), Barczy and Pap (2010, 2011) have obtained the different asymptotic behavior of $\hat{\alpha}_t$. Moreover, Jiang (2009) studied the moderate deviations for this estimator. In this paper, we aim to derive the Berry–Esseen bound of $\hat{\alpha}_t$. Furthermore, we can apply the Berry–Esseen bound to consider the precise asymptotics in the law of iterated logarithm (LIL) for $\hat{\alpha}_t$.

Our main result is the following theorem.

Theorem 1.1. *Suppose*

$$\operatorname{sign}(\alpha - K) = \operatorname{sign} K, \quad K \neq 0 \tag{1.3}$$

and

$$\int_0^T \sigma^2(t)\mathrm{d}t < +\infty, \qquad b(t) = \frac{\sigma^2(t)}{-2K\int_t^T \sigma^2(s)\mathrm{d}s}. \tag{1.4}$$

Then

$$\sup_{x\in\mathbb{R}}\left|P\left(\left(\frac{\log\int_t^T \sigma^2(s)\mathrm{d}s}{4K(K-\alpha)}\right)^{\frac{1}{2}}(\hat{\alpha}_t - \alpha) \le x\right) - \Phi(x)\right| = O\left(\left(-\log\int_t^T \sigma^2(s)\mathrm{d}s\right)^{-\frac{1}{2}}\right),$$

where $\Phi(x)$ is the standard normal distribution function. □

Remark 1.1. As $t \uparrow T$, we can see that $(-\log\int_t^T \sigma^2(s)\mathrm{d}s)^{\frac{1}{2}} \uparrow +\infty$.

2. Berry–Esseen bound

Throughout the paper, C denotes a generic positive constant (perhaps depending on α, but not on anything else). It is easy to see that the Eq. (1.1) has a unique strong solution

$$X_t^{(\alpha)} = \int_0^t \sigma(s)\mathrm{e}^{\alpha\int_s^t b(u)\mathrm{d}u}\mathrm{d}W_s, \quad t \in [0, T). \tag{2.1}$$

By simple calculation, we can obtain the following result from Theorem 2.2 in Barczy and Pap (2011).

Lemma 2.1. *For $z_1, z_2 \in \mathbb{C}$, let*

$$\phi_t(z_1, z_2) = E\exp\left(z_1\int_0^t \frac{b^2(s)}{\sigma^2(s)}(X_s^{(\alpha)})^2\mathrm{d}s + z_2(X_t^{(\alpha)})^2\right).$$

Then $\phi_t(z_1, z_2)$ exists for $|z_i| < \delta,\ i = 1, 2$ for some $\delta > 0$ and

$$\phi_t(z_1, z_2) = \mathrm{e}^{\frac{K-\alpha}{4K}\log\frac{\int_t^T \sigma^2(s)\mathrm{d}s}{\int_0^T \sigma^2(s)\mathrm{d}s}}\left(\frac{2\gamma}{\hat{\phi}_t(z_1, z_2) + \bar{\phi}_t(z_1, z_2)}\right)^{\frac{1}{2}},$$

where

$$\hat{\phi}_t(z_1, z_2) = \left(\gamma + \alpha - K - 4Kz_2\int_t^T \sigma^2(s)\mathrm{d}s\right)\mathrm{e}^{-\frac{\gamma}{2K}\log\frac{\int_t^T \sigma^2(s)\mathrm{d}s}{\int_0^T \sigma^2(s)\mathrm{d}s}},$$

$$\bar{\phi}_t(z_1, z_2) = \left(\gamma - (\alpha - K) + 4Kz_2\int_t^T \sigma^2(s)\mathrm{d}s\right)\mathrm{e}^{\frac{\gamma}{2K}\log\frac{\int_t^T \sigma^2(s)\mathrm{d}s}{\int_0^T \sigma^2(s)\mathrm{d}s}}$$

with $\gamma = \sqrt{(\alpha - K)^2 - 2z_1}$ and we always choose the principal branch of the square root. □

Lemma 2.2. (a) *Let*

$$G_{t,x} := \left(\frac{4K(K-\alpha)}{\log\int_t^T \sigma^2(s)\mathrm{d}s}\right)^{\frac{1}{2}}\int_0^t \frac{b(s)}{\sigma(s)}X_s^{(\alpha)}\mathrm{d}W_s - \left(\frac{4K(K-\alpha)}{\log\int_t^T \sigma^2(s)\mathrm{d}s}\int_0^t \frac{b^2(s)}{\sigma^2(s)}(X_s^{(\alpha)})^2\mathrm{d}s - 1\right)x. \tag{2.2}$$