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System availability behavior of some stationary dependent sequences

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A B S T R A C T

We consider the system availability behavior of a one-unit repairable system when the failure and the repair times are generated by a stationary dependent sequence of random variables. We obtain the general expression for the point availability, and discuss the nature of the availability measure for two time series models: a first-order exponential moving average process and a first-order exponential autoregressive process.

1. Introduction

Consider a one-unit repairable system which is at any time in one of two states, up or down. We can interpret the up state as the system being functioning and the down state as the system being not functioning. If we define $\xi(t)$ as the state of the system at time t, we have

$$\xi(t) = \begin{cases} 1 & \text{if the system is operating at time } t \\ 0 & \text{otherwise.} \end{cases}$$

Based on $\xi(t)$, a number of useful measures of the system availability may be constructed. The point availability, $A(t) = P[\xi(t) = 1]$, and the limiting availability, $A = \lim_{t\to\infty} A(t)$, are two commonly used measures. The point availability is defined as the probability that the system is operational at a given point in time. Since it is difficult to obtain an explicit expression for the point availability except for few simple cases, in the literature more attention is being paid to its limiting measure A. The limiting availability measure is useful when one may be interested in knowing the extent to which the system will be available after it has been run for a long time. The properties of these two measures are usually studied using the successive failure and repair times of the system.

One of the major limitations in the study of system availability is the assumption of independence among the successive sequence of failure and repair times of the system. When the system is operating in a random environment, it is natural to expect some sort of dependence among the successive sequence of failure and repair times. So, it is important to consider suitable models for repairable systems that can incorporate the dependence structure. Motivated by this idea, Kijima and Sumita (1986) discussed point process models for the reliability of repairable systems when the survival times are generated by stationary dependent sequences. Several non-Gaussian time series models such as first-order random coefficient autoregressive models are discussed in the literature for modeling life time data. See, for example, Lawrance and

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Lewis (1977), Gaver and Lewis (1980), and Sim (1992). In the case of repairable systems, the study of availability measures for stationary dependent sequences is not discussed much, except those considered by Abraham and Balakrishna (2000) and Balakrishna and Mathew (2012). In this paper, we study the system availability behavior of a repairable system when the failure and repair times are generated by some time series models with exponential marginal distributions.

2. Point availability of stationary dependent sequences

Let $\{X_n\}$ and $\{Y_n\}$ be two independent sequences of stationary dependent non-negative random variables with distribution functions $F_X(.)$ and $F_Y(.)$, respectively. Suppose that U_n denotes the sum of the first n operating intervals such that $U_n = \sum_{i=1}^{n} X_i$, and let $F_X^{(n)}(.)$ be the distribution function of U_n. Let $V_n = \sum_{i=1}^{n} Y_i$ be the sum of the first n repair intervals, and let $F_Y^{(n)}(.)$ be the distribution function of V_n. To capture the behavior of cycles that contain one operation interval and one repair interval, it is useful to define $Z_n = X_n + Y_n$ as the length of the nth such cycle. Let S_n be the total elapsed time of the first n cycles such that $S_n = \sum_{i=1}^{n} Z_i = U_n + V_n$, and let $F_Z^{(n)}(.)$ be the distribution function of S_n so that $F_Z^{(n)}(t) = F_X^{(n)} * F_Y^{(n)}(t)$, where '$*$' denotes the convolution operator.

Now, the point availability of the repairable system is given by

$$A(t) = P[\xi(t) = 1] = \bar{F}_X(t) + \sum_{n=1}^{\infty} \int_0^t P[X_{n+1} > t - u \,|S_n = u] dF_Z^{(n)}(u). \tag{1}$$

The first term in the point availability function (1) reflects the probability that the first period of operation is of length t or greater. The subsequent integral expressions reflect the probability that the nth failure occurs at time u and the following period of operation is of length $(t-u)$ or greater.

We can write

$$\begin{aligned} \int_0^t P[X_{n+1} > t - u \,|S_n = u] dF_Z^{(n)}(u) &= \int_0^t dF_Z^{(n)}(u) - \int_0^t P[X_{n+1} \le t - u \,|S_n = u] dF_Z^{(n)}(u) \\ &= F_Z^{(n)}(t) - P[X_{n+1} + S_n \le t] = F_Z^{(n)}(t) - P[U_{n+1} + V_n \le t] \\ &= F_X^{(n)} * F_Y^{(n)}(t) - F_X^{(n+1)} * F_Y^{(n)}(t) = \left(F_X^{(n)} - F_X^{(n+1)}\right) * F_Y^{(n)}(t). \end{aligned}$$

Thus the point availability can be expressed as

$$A(t) = \bar{F}_X(t) + \sum_{n=1}^{\infty} \left(F_X^{(n)} - F_X^{(n+1)}\right) * F_Y^{(n)}(t). \tag{2}$$

The Laplace transform of the point availability function $A(t)$ is

$$A^*(s) = \bar{F}_X^*(s) + \sum_{n=1}^{\infty} \left(F_X^{(n)*}(s) - F_X^{(n+1)*}(s)\right) F_Y^{(n)*}(s), \tag{3}$$

where $F_X^{(n)*}(s)$ and $F_Y^{(n)*}(s)$ denote the Laplace transforms of $F_X^{(n)}(t)$ and $F_Y^{(n)}(t)$, respectively.

If the Laplace transforms $F_X^{(n)*}(s)$ and $F_Y^{(n)*}(s)$ are known, then the point availability function can be obtained by inverting the Laplace transform $A^*(s)$ given in (3).

Note: If we assume that $\{X_n\}$ and $\{Y_n\}$ are two independent sequences of i.i.d. non-negative random variables, then $F_X^{(n+1)}(t) = F_X * F_X^{(n)}(t)$, and (2) reduces to $A(t) = \bar{F}_X(t) + \bar{F}_X * M(t)$, where $M(t) = \sum_{n=1}^{\infty} F_X^{(n)} * F_Y^{(n)}(t)$ is the renewal function associated with the sequence $\{Z_n\}$.

3. Availability behavior of first-order exponential moving-average processes

Assume that the sequences of failure and repair times, $\{X_n\}$ and $\{Y_n\}$, are generated by two independent first-order exponential moving average (EMA1) processes (Lawrance and Lewis, 1977) defined as

$$X_n = \begin{cases} \beta_1 \varepsilon_n & \text{with probability } \beta_1, \\ \beta_1 \varepsilon_n + \varepsilon_{n+1} & \text{with probability } 1 - \beta_1 \end{cases} \quad \text{and}$$

$$Y_n = \begin{cases} \beta_2 \eta_n & \text{with probability } \beta_2, \\ \beta_2 \eta_n + \eta_{n+1} & \text{with probability } 1 - \beta_2, \end{cases} \quad (0 \le \beta_1, \beta_2 \le 1, n = 1, 2, 3, \ldots),$$

where $\{\varepsilon_n\}$ and $\{\eta_n\}$ are two independent i.i.d. exponential random sequences with parameters λ_1 and λ_2, respectively. The simplest aspects of the EMA1 model are the exponential marginal distribution of the intervals and the non-Markovian dependence among the adjacent members of the sequence.