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Optimal financing and dividend control of the insurance company with excess-of-loss reinsurance policy[☆]

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ABSTRACT

In this paper, we consider an optimal financing and dividend control problem of an insurance company. The management of the insurance company controls the dividends payout, equity issuance and the excess-of-loss reinsurance policy. In our model, the dividends are assumed to be paid out continuously, which is of interest from the perspective of financial modeling. The objective is to find the strategy which maximizes the expected present values of the dividends payout minus the equity issuance up to the time of ruin. We solve the optimal control problem and identify the optimal strategy by constructing two categories of suboptimal control problems.

1. Introduction

This paper is concerned with the optimal financing and dividend control problem of an insurance company. We assume that the company can control its reserves by paying dividends, issuing equity and taking reinsurance. In this model, we study the case of excess-of-loss reinsurance for an insurance company. The objective of the company is to maximize the expected present values of the dividends payout minus the equity issuance up to the time of ruin.

Diffusion models for companies with controllable risk exposure and dividend payment have been extensively studied in the literature. For example, see Asmussen et al. (2000), Højgaard and Taksar (1999, 2001, 2004), Choulli et al. (2001), Cadenillas et al. (2006) and the references therein. In these papers, the authors assumed that the company reduces its risk exposure by proportional reinsurance.

Sethi and Taksar (2002) considered a model for the company that can control its risk exposure by issuing new equity as well as by paying dividends. Løkka and Zervos (2008) considered the same problem with the possible bankruptcy. He and Liang (2008) studied an optimal financing and dividend control problem of an insurance company with proportional reinsurance policy. In these papers, it was assumed that the dividends are paid out continuously. He and Liang (2009) studied the case where the dividends are paid out at discrete random times with fixed and proportional transaction costs.

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A natural question is how about the optimal excess-of-loss reinsurance policy for the case where the dividends are paid out continuously. In the present paper, we will solve this problem, and give the optimal excess-of-loss reinsurance policy, dividend payout and equity issuance policy.

It should be mentioned that Meng and Siu (2011) have studied a similar optimal financing and dividend control problem of an insurance company, where the dividends are paid out at discrete random times with fixed and proportional transaction costs. In the present paper, we do not take into account the fixed and proportional transaction costs, but consider the case where the dividends are paid out continuously. Like those references mentioned in the third paragraph, establishing a model in which the dividends are paid out continuously is of theoretical interest from the perspective of financial modeling. The main differences between the present paper and the paper by Meng and Siu (2011) are the ways to construct the solutions to the corresponding HJB equations. In the present paper, we have to show the differentiability of the solution of the HJB equation. Meng and Siu (2011) considered the viscosity solution of the HJB equation. In this sense, the present paper can be considered as the counterpart of Meng and Siu (2011).

The rest of the paper is organized as follows. In Section 2, we state the formulation of the problem. In Section 3, we review the solution to the control problem without any equity issuance. In Section 4, we solve the control problem for a company that does not allow for bankruptcy by equity issuance. In Section 5, we identify the value function and the optimal strategy with the corresponding solution in either category of suboptimal models.

2. Problem formulation

To specify our diffusion model, we start with the classical Cramer–Lundberg risk model. In this model, the reserve of the company without any reinsurance and dividend payments is defined by

$$R_t = R_0 + pt - \sum_{i=1}^{N_t} U_i, \quad t > 0, \tag{2.1}$$

where $p > 0$ is the premium rate, $\{N_t;\ t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$ and $\{U_i;\ i \geq 1\}$ are i.i.d. positive random variables with common distribution F having finite first two moments. The premium rate is given by

$$p = (1+\eta)\lambda E(U_1), \tag{2.2}$$

where $\eta > 0$ is the relative safety loading.

Let a be the (fixed) excess-of-loss retention level and $U_i^{(a)}$ be the size of the ith claim held by the insurer. Assume that the safety loading for the reinsurer is the same as that for the insurer. Then the reserve process of the insurer is given by

$$R_t^{(a,\eta)} = u + p^{(a,\eta)}t - \sum_{i=1}^{N_t} U_i^{(a)}, \tag{2.3}$$

where $U_i^{(a)} = U_i \wedge a$ and $p^{(a,\eta)} = (1+\eta)\lambda E(U_1^{(a)})$. Without loss of generality, we assume $\lambda = 1$. By the same argument as in Asmussen et al. (2000), we can get the following diffusion approximation model with excess-of-loss reinsurance

$$dR_t = \mu(a)dt + \sigma(a)dB_t, \tag{2.4}$$

where $\{B_t;\ t \geq 0\}$ is a standard Brownian motion, and

$$\mu(a) := E(U_i^{(a)}) = \int_0^a \bar{F}(x)dx, \tag{2.5}$$

$$\sigma^2(a) := E[(U_i^{(a)})^2] = \int_0^a 2x\bar{F}(x)dx, \tag{2.6}$$

where $\bar{F}(x) := P(U_i > x) = 1 - F(x)$. Define

$$N := \inf\{x \geq 0 : \bar{F}(x) = 0\}. \tag{2.7}$$

Then both functions $\mu(\cdot)$ and $\sigma^2(\cdot)$ are increasing on $[0, N]$, while on $[N, \infty)$ they are constants equal to $\mu_\infty := \mu(N)$ and $\sigma^2_\infty := \sigma^2(N)$.

Let us begin with a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ endowed with the information filtration $\{\mathcal{F}_t;\ t \geq 0\}$, and a process $\{B(t); t \geq 0\}$ which is a standard Brownian motion adapted to $\{\mathcal{F}_t;\ t \geq 0\}$. We consider the retention level to be a dynamically controlled parameter. At each time $t \geq 0$, the retention level $a = a(t)$ is chosen by the insurance company. We assume that $a(0) = 0$, which is reasonable. In addition, we denote by L_t the cumulative amount of dividends paid out up to time t, and by G_t the total amount raised by issuing equity up to time t.

A policy $\pi := (a^\pi, L^\pi, G^\pi)$ is a triple of $\{\mathcal{F}_t\}$-adapted stochastic process $\{(a_t^\pi, L_t^\pi, G_t^\pi);\ t \geq 0\}$. A policy π is called admissible policies if $0 \leq a_t^\pi < +\infty$ for all $t \geq 0$, $\{L_t^\pi;\ \geq 0\}$ and $\{G_t^\pi;\ \geq 0\}$ are increasing and right-continuous with left limits, where $L_0 := 0$ and $G_0 := 0$. We denote the set of all admissible policies by Π.