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Rates of convergence of extreme for asymmetric normal distribution☆

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ABSTRACT

In this paper, we derive the exact uniform convergence rate of the asymmetric normal distribution of the maximum and minimum to its extreme value limit.

1. Introduction

Let $\{Y_n,\ n \geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) random variables with common distribution function $F(x)$, let $M_n = \max\{Y_1, \ldots, Y_n\}$ denote the partial maximum. If there exist normalizing constants $a_n > 0$, $b_n \in R$ and non-degenerate distribution $G(x)$ such that

$$\lim_{n\to\infty} P(M_n \leq a_n x + b_n) = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x) \tag{1.1}$$

for all continuity points of G, then G must belong to one of the following three classes:

$$\Phi_\alpha(x) = \begin{cases} 0, & x < 0, \\ \exp\{-x^{-\alpha}\}, & x \geq 0, \end{cases}$$

$$\Psi_\alpha(x) = \begin{cases} \exp\{-(-x)^{\alpha}\}, & x < 0, \\ 1, & x \geq 0, \end{cases}$$

for some $\alpha > 0$ and

$$\Lambda(x) = \exp\{-e^{-x}\}, \quad x \in R.$$

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If (1.1) holds, we say that F belongs to one of the max domains of attraction of G, denoted by $F \in D_{\max}(G)$. Criteria for $F \in D_{\max}(G)$ and the choice of normalizing constants, a_n and b_n, can be found in de Haan (1970), Galambos (1987), Leadbetter et al. (1983) and Resnick (1987).

Similarly, let $W_n = \min\{Y_1, \ldots, Y_n\}$ denote the partial minimum, there must exist normalizing constants $c_n > 0,\ d_n \in R$ and non-degenerate distribution $L(x)$ such that

$$\lim_{n\to\infty} P(W_n \le c_n x + d_n) = \lim_{n\to\infty} \{1 - [1 - F(c_n x + d_n)]^n\} = L(x) \tag{1.2}$$

for all continuity points of $L(x)$, then L must belong to one of the following three classes:

$$L_1(x) = \begin{cases} 1 - \exp\{-(-x)^{-\alpha}\}, & x < 0, \\ 1, & x \ge 0, \end{cases}$$

$$L_2(x) = \begin{cases} 0, & x < 0, \\ 1 - \exp\{-x^{\alpha}\}, & x \ge 0, \end{cases}$$

for some $\alpha > 0$ and

$$L_3(x) = 1 - \exp\{-e^x\}, \quad x \in R.$$

If (1.2) holds, we say that F belongs to one of the min domains of attraction of L, denoted by $F \in D_{\min}(L)$. Criteria for $F \in D_{\min}(L)$ and the choice of normalizing constants, c_n and d_n, can be found in Galambos (1987).

One interesting problem in extreme value is the convergence rate of $F^n(a_n x + b_n)$ to any one of the extreme value distributions. For the uniform convergence rate of $F^n(a_n x + b_n)$ to its extreme value limit, $\Lambda(x)$, Hall and Wellner (1979) showed that the convergence rate is proportional to $1/n$ if F is exponential. For the normal distribution, Hall (1979) proved the following result:

$$\frac{c_1}{\log n} < \sup_{x\in R} |\Phi^n(a_n x + b_n) - \Lambda(x)| < \frac{c_2}{\log n}$$

for $n > n_0$, where constants $0 < c_1 < c_2$ and $\Phi(x)$ denotes the standard normal distribution function, and the norming constants a_n and b_n are given by

$$2\pi b_n^2 \exp(b_n^2) = n^2, \qquad a_n = b_n^{-1}.$$

Hall (1979) showed that $1/\log n$ is the best convergence rate for the maxima of normal random variables. For related work on the uniform convergence rate of extremes, see Smith (1982), Falk (1986) and Kaufmann (1995). For general distribution functions, de Haan and Resnick (1996) gave the rate of uniform convergence under a second order regular variation condition involved in a distribution function F with second derivative F'' which was weakened to the case of F without any derivative requirement in Cheng and Jiang (2001). For recent work, see Peng et al. (2010), Lin et al. (2011), Liao and Peng (2012) and Liao et al. (2012): Peng et al. (2010) derived convergence rates of the distribution of maxima for random sequences obeying the general error distribution; Lin et al. (2011) obtained convergence rates of the distribution of maxima for random sequences obeying the short-tailed symmetric distribution; Liao and Peng (2012) discussed convergence rates of the distribution of maxima for random sequences obeying the lognormal distribution; Liao et al. (2012) established two different convergence rates of the distribution of maxima for random sequences obeying the skew normal distribution. Our aim in this paper is to consider the uniform convergence rate of (1.1) and (1.2) when X_n follows the asymmetric normal distribution ($\mathcal{AN}$ distribution). The $\mathcal{AN}$ distribution is one of the most widely applied distributions in statistics (Alan and Pond, 1988; Aigner et al., 1977; George and Greg, 1977; Micceri, 1989). For example, Kato et al. (2002) apply it to pattern recognition. The asymmetric normal distribution has attracted applications in subsidence analysis and training and overtraining markers in sport events. Some references are Johnson et al. (1994), Azzalini and Dalla (1996) and Bennett (2003). The probability density function (pdf) is given by:

$$F'(x) = \frac{2}{\sqrt{2\pi}(\sigma_l + \sigma_r)} \begin{cases} \exp\left\{-\dfrac{(x-\theta)^2}{2\sigma_l^2}\right\}, & x \le \theta, \\ \exp\left\{-\dfrac{(x-\theta)^2}{2\sigma_r^2}\right\}, & x > \theta, \end{cases} \tag{1.3}$$

where $\sigma_l > 0,\ \sigma_r > 0$ and $\theta \in R$.

This paper is organized as follows: a lemma and some auxiliary results are given in Section 2. Main results are given in Section 3. Their proofs are deferred to Section 4.

2. Preliminaries

In order to obtain the uniform convergence rate of extremes from the $\mathcal{AN}$ distribution, we will give a lemma and two propositions. The following lemma gives Mills-type ratios for the $\mathcal{AN}$ distribution.