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A note on shrinkage wavelet estimation in Bayesian analysis

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ABSTRACT

In this short note the closed form of the soft wavelet shrinkage estimator is derived, extending the work of Huang (2002) for the scale mixture of normal distributions.
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1. Introduction

Following Huang (2002) consider the discrete noisy signal model

$$w = \theta + \epsilon,$$

where $w = (w_1, \ldots, w_n)^T$ is the vector of empirical wavelet coefficients, $\epsilon = (\epsilon_1, \ldots, \epsilon_n)^T$ is the vector of disturbances with zero mean and common variance σ^2, and $\theta = (\theta_1, \ldots, \theta_n)^T$ is the vector of true wavelet coefficients. Let $\delta(w) = \big(\delta_1(w), \ldots, \delta_n(w)\big)^T$ be an estimator of θ. The soft thresholding wavelet shrinkage estimation by Donoho and Johnstone (1994) is given by

$$\delta_i^{\text{soft}}(w, \lambda) = sign(w_i)(|w_i| - \lambda)I(|w_i| \geq \lambda), \quad i = 1, \ldots, n, \tag{1}$$

where $I(.)$ is an indicator function and $\lambda > 0$ is a threshold parameter. Huang (2002) investigated the shrinkage wavelet estimation problem in the multivariate normal model. To extend his result for a general type of distributions, a similar problem can be treated via the estimation of the mean vector θ for a scale mixture of multivariate normal distributions. In this regard, we employ the asymmetric LINEX loss function (Varian, 1975; Zellner, 1986) as the error criterion. Under such a loss function, we derive the closed form of a generalized Bayes estimator, which is also shown to be the unique admissible and minimax estimator. Then, we show that the soft wavelet shrinkage estimator (1) can be derived as an empirical version of the admissible and minimax generalized Bayes estimator.

For a precise error setup, assume that the error vector ϵ, follows the scale mixture of multivariate normal distributions as

$$f(\epsilon) = \int_0^\infty (2\pi)^{-\frac{1}{2}}\, t^{\frac{1}{2}} \exp\left(-\frac{1}{2}tz^2\right) \mathrm{d}G(t), \tag{2}$$

where $Z^2 = \epsilon^2/\sigma^2$ and t is a positive random variable, a mixture dynamic component with distribution G. We will use the notation $\epsilon \sim \mathcal{SMN}(0, \sigma^2; G)$. Some distributions in this class are the multivariate t, slash and contaminated normal

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distributions. It is convenient to write the distribution of ϵ alternatively using the following hierarchical representation

$$\epsilon|t \sim \mathcal{N}(0, t^{-1}\sigma^2), \quad t \sim G(t; v). \tag{3}$$

This class of distributions represents an interesting alternative to the normal in the presence of outliers (see Lange and Sinsheimer, 1993). From the above assumption, it is readily deduced that

$$w \sim \mathcal{SMN}_n(\theta, \sigma^2, G). \tag{4}$$

2. Loss function

In many real life situations over-estimation and under-estimation of the same magnitude have different importance and actually different economic and physical implications. In this condition, it is inappropriate to use a symmetric loss function such as the squared error, because of the failure to differentiate between over and under estimation, so an appropriate loss function is asymmetric.

Under the assumptions of Section 1, consider the following linear exponential (LINEX) loss function for δ with the scale parameter b and the shape parameter a_i:

$$\mathcal{L}(\Delta) = b\sum_{i=1}^{n}\left(e^{a_i\Delta_i} - a_i\Delta_i - 1\right), \quad a_i \neq 0,\ b > 0,\ i = 1, \ldots, n, \tag{5}$$

where $\Delta_i = \delta_i(w) - \theta_i$ denotes the estimation error in using $\delta_i(w)$ to estimate θ_i. We see that, for a negative a_i, the following term:

$$e^{a_i\Delta_i} - a_i\Delta_i - 1 \tag{6}$$

rises almost exponentially when $\Delta_i < 0$ and almost linearly when $\Delta_i > 0$. For a positive a_i the situation is reversed. Thus, the negativity of a_i, discourages under-estimation and the positivity of a_i, discourages over-estimation. For small values of $|a_i|$, the function is almost symmetric and not far from a squared loss function (see Zellner, 1986).

3. Main results

In this section we propose a shrinkage generalized Bayes (GB) wavelet estimator and prove its admissible and minimaxity.

Theorem 3.1. *Under the loss (5), assuming $b = \frac{1}{n}$, the estimator $\delta^{\mathrm{GB}}(w)$ given by*

$$\delta_i^{\mathrm{GB}} = w_i - \frac{\ln\alpha(a_i^2, \sigma^2)}{a_i}, \quad i = 1, \ldots, n \tag{7}$$

where

$$\alpha(a_i^2, \sigma^2) = \int_0^\infty e^{\frac{a_i^2\sigma^2}{2t}}\, \mathrm{d}G(t), \quad i = 1, \ldots, n,$$

is a soft wavelet shrinkage estimator for θ with respect to the flat improper prior $\pi(\theta) = 1, \theta \in \mathbb{R}^n$.

Proof. First we prove the estimator given by (7) is a GB estimator. The posterior distribution is

$$\pi(\theta|w) \propto \pi(\theta)f(w|\theta) \sim \mathcal{SMN}_n(w, \sigma^2, G).$$

It is straightforward to check that the posterior expected loss of an arbitrary estimator $\delta(w)$ is given by

$$\begin{aligned}\rho(\pi(\theta|w), \delta(w)) &= \int_\Theta \mathcal{L}(\theta, \delta(w))\, \mathrm{d}\pi(\theta|w) \\ &= \frac{1}{n}\int_0^\infty \sum_{i=1}^{n}\left[e^{a_i(\delta_i - w_i) + \frac{a_i^2\sigma^2}{2t}} g(t)dt - a_i(\delta_i - w_i) - 1\right] \\ &= \frac{1}{n}\sum_{i=1}^{n}\left[e^{a_i(\delta_i - wi)} \times \alpha(a_i^2, \sigma^2)_{-}a_i(\delta_i - w_i) - 1\right].\end{aligned} \tag{8}$$

A generalized Bayes estimator is an estimator δ which minimizes (8). First, we take derivatives with respect to δ_i and set them to zero. We get a system of equations

$$a_i e^{a_i(\delta_i - w_i)} \times \alpha(a_i^2, \sigma^2) - a_i = 0, \quad i = 1, \ldots, n$$

or equivalently,

$$\ln\alpha(a_i^2, \sigma^2) + a_i(\delta_i - w_i) = 0, \quad i = 1, \ldots, n.$$

The unique solution for the above system of equations, which is the generalized Bayes estimator with respect to the flat prior over $\mathbb{R}^n$, is given by (7).

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