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Complete moment convergence of moving average processes under φ-mixing assumptions

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ABSTRACT

Let $\{Y_i : -\infty < i < \infty\}$ be a sequence of identically distributed φ-mixing random variables, and $\{a_i : -\infty < i < \infty\}$ an absolutely summable sequence of real numbers. In this work we prove the complete moment convergence for the partial sums of moving average processes $\{X_n = \sum_{i=-\infty}^{\infty} a_i Y_{i+n} : n \geq 1\}$, improving the result of [Kim, T.S., Ko, M.H., 2008. Complete moment convergence of moving average processes under dependence assumptions. Statist. Probab. Lett. 78, 839–846].

1. Introduction

We assume that $\{Y_i : -\infty < i < \infty\}$ is a sequence of identically distributed random variables. Let $\{a_i : -\infty < i < \infty\}$ be an absolutely summable sequence of real numbers, and set, for $n \geq 1$,

$$X_n = \sum_{i=-\infty}^{\infty} a_i Y_{i+n}. \tag{1.1}$$

The limiting behavior of the moving average process $\{X_n : n \geq 1\}$ has been extensively investigated when $\{Y_i : -\infty < i < \infty\}$ is a sequence of independent random variables (see, e.g., Ibragimov (1962), Burton and Dehling (1990), Li et al. (1992)). The more general case where $\{Y_i : -\infty < i < \infty\}$ is φ-mixing has been considered by Zhang (1996) (see also Shao (1988), Yu and Wang (2002), Baek et al. (2003), Li and Zhang (2004), Kim and Ko (2008) and Chen et al. (2009)). The following Theorems A, B1–B2 and C are due, respectively, to Zhang (1996), Chen et al. (2009), and Kim and Ko (2008).

Throughout the sequel, h will denote a real valued, positive and measurable function on $\mathbb{R}^+$.

Theorem A. *Let h be a function slowly varying at infinity, $1 \leq p < 2$, and $r \geq 1$. Suppose that $\{X_n : n \geq 1\}$ is a moving average process based on a sequence $\{Y_i : -\infty < i < \infty\}$ of identically distributed φ-mixing random variables with $\sum_{m=1}^{\infty} \varphi^{1/2}(m) < \infty$. If $EY_1 = 0$ and $E|Y_1|^{rp} h(|Y_1|^p) < \infty$, then*

$$\sum_{n=1}^{\infty} n^{r-2} h(n) P\left\{\left|\sum_{k=1}^{n} X_k\right| \geq \varepsilon n^{1/p}\right\} < \infty, \quad \textit{for all } \varepsilon > 0.$$

Theorem B1. *Let h be a function slowly varying at infinity, $1 \leq p < 2$, and $r > 1$. Suppose that $\{X_n : n \geq 1\}$ is a moving average process based on a sequence $\{Y_i : -\infty < i < \infty\}$ of identically distributed φ-mixing random variables. If $EY_1 = 0$ and*

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$E|Y_1|^{rp}h(|Y_1|^p) < \infty$, *then*

$$\sum_{n=1}^{\infty} n^{r-2}h(n)P\left\{\max_{1\le k\le n}\left|\sum_{j=1}^{k}X_j\right| \ge \varepsilon n^{1/p}\right\} < \infty, \quad \textit{for all } \varepsilon > 0$$

and

$$\sum_{n=1}^{\infty} n^{r-2}h(n)P\left\{\sup_{k\ge n}\left|k^{-\frac{1}{p}}\sum_{j=1}^{k}X_j\right| \ge \varepsilon\right\} < \infty, \quad \textit{for all } \varepsilon > 0.$$

Theorem B2. *Let h be a function slowly varying at infinity and* $1 \le p < 2$. *Assume that* $\sum_{i=-\infty}^{\infty}|a_i|^{\theta} < \infty$, *where* θ *belong to* $(0, 1)$ *if* $p = 1$ *and* $\theta = 1$ *if* $1 < p < 2$. *Suppose that* $\{X_n : n \ge 1\}$ *is a moving average process based on a sequence* $\{Y_i : -\infty < i < \infty\}$ *of identically distributed* φ*-mixing random variables with* $\sum_{m=1}^{\infty}\varphi^{1/2}(m) < \infty$. *If* $EY_1 = 0$ *and* $E|Y_1|^p h(|Y_1|^p) < \infty$, *then*

$$\sum_{n=1}^{\infty} \frac{h(n)}{n}P\left\{\max_{1\le k\le n}\left|\sum_{j=1}^{k}X_j\right| \ge \varepsilon n^{1/p}\right\} < \infty, \quad \textit{for all } \varepsilon > 0.$$

Theorem C. *Let h be a function slowly varying at infinity,* $1 \le p < 2$, *and* $r \ge 1 + p/2$. *Suppose that* $\{X_k : k \ge 1\}$ *is a moving average process based on a sequence* $\{Y_i : -\infty < i < \infty\}$ *of identically distributed* φ*-mixing random variables with* $EY_1 = 0$, $EY_1^2 < \infty$ *and* $\sum_{m=1}^{\infty}\varphi^{1/2}(m) < \infty$. *If* $E|Y_1|^{rp}h(|Y_1|^p) < \infty$, *then*

$$\sum_{n=1}^{\infty} n^{r-2-1/p}h(n)E\left\{\left|\sum_{k=1}^{n}X_k\right| - \varepsilon n^{1/p}\right\}^{+} < \infty, \quad \textit{for all } \varepsilon > 0.$$

In this work, we shall extend Theorems B1 and B2 to the framework of complete moment convergence. Our methods differ from those used by Kim and Ko (2008), and our results improve upon their Theorem C. Below, $C, C_1, C_2, \ldots$ will denote generic positive constants, whose value may vary from one application to another, $I[A]$ will indicate the indicator function of A, and we will set $\lfloor x\rfloor \le x < \lfloor x\rfloor + 1$ for the integer part of x.

2. Main results and some lemmas

We suppose that $\{Y_i : -\infty < i < \infty\}$ is a sequence of identically distributed and φ-mixing random variables, i.e.,

$$\varphi(m) = \sup_{k\ge 1}\sup\{|P(B|A) - P(A)|, A \in \mathcal{F}_{-\infty}^{k}, P(A) \ne 0, B \in \mathcal{F}_{k+m}^{\infty}\} \to 0$$

as $m \to \infty$, where $\mathcal{F}_n^m = \sigma(Y_i, n \le i \le m)$, $-\infty \le n \le m \le \infty$.

Recall that a real valued function h, positive and measurable on $\mathbb{R}^+$, is said to be slowing varying if for each $\lambda > 0$

$$\lim_{x\to\infty}\frac{h(\lambda x)}{h(x)} = 1.$$

Now we state our main results. The proofs will be given in Section 3.

Theorem 2.1. *Let h be a function slowly varying at infinity,* $1 \le p < 2$, *and* $r > 1$. *Suppose that* $\{X_n : n \ge 1\}$ *is a moving average process based on a sequence* $\{Y_i : -\infty < i < \infty\}$ *of identically distributed* φ*-mixing random variables. If* $EY_1 = 0$ *and* $E|Y_1|^{rp}h(|Y_1|^p) < \infty$, *then*

$$\sum_{n=1}^{\infty} n^{r-2-1/p}h(n)E\left\{\max_{1\le k\le n}\left|\sum_{j=1}^{k}X_j\right| - \varepsilon n^{1/p}\right\}^{+} < \infty, \quad \textit{for all } \varepsilon > 0 \tag{2.1}$$

and

$$\sum_{n=1}^{\infty} n^{r-2}h(n)E\left\{\sup_{k\ge n}\left|k^{-\frac{1}{p}}\sum_{j=1}^{k}X_j\right| - \varepsilon\right\}^{+} < \infty, \quad \textit{for all } \varepsilon > 0. \tag{2.2}$$