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Uniqueness, recurrence and decay properties of collision branching processes with immigration

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A B S T R A C T

We consider a kind of Collision Branching Processes with Immigration (CBIP). Some important properties of the generating functions for the CBI q-matrix were first investigated in detail. Then for any given CBI q-matrix, the existence and uniqueness of CBIP is proved, and sufficient and easily checked conditions for the CBIP to be recurrent are given. Moreover, the exact value of the decay parameter λ_Z is obtained and expressed explicitly for the communicating class $\mathbb{Z}_+$ in the case that the immigration is independent of states. It is shown that this λ_Z can be directly obtained from the generating functions of the corresponding q-matrix. Finally, the invariant vectors and invariant measures are considered.

1. Introduction

We mainly consider the recurrence property and decay property of collision branching processes with immigration, such as the decay parameter, invariant measure/vector and equilibrium distribution. The evolution of the particles in the system can be described as follows.

(i) Only collisions between two particles will give "offspring". Collisions occur at random and, whenever two particles collide, they are removed and replaced by j "offsprings" with probability p_j $(j \geq 0)$, independently of other collisions.
(ii) Immigration will occur at any state.
(iii) Immigrants from outside will follow the same reproductive rules as above.

We now begin to specify the model as a continuous time Markov chain with the state space $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$. In order that the branching property holds for the ordinary Markov branching process, it is necessary that its transition function obeys the Kolmogorov forward equations. Guided by this fact, we formally define the collision branching process with immigration as follows.

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Definition 1.1. A q-matrix $Q = (q_{ij}; i, j \in \mathbb{Z}_+)$ is called a collision branching q-matrix with immigration (henceforth referred to as a CBI q-matrix) if it takes the following form.

$$q_{ij} = \begin{cases} h_j & \text{if } i = 0,\ j \ge 0 \\ a_{j-1} & \text{if } i = 1,\ j \ge 1 \\ \binom{i}{2} b_{j-i+2} & \text{if } i \ge 2,\ j = i-1, i-2 \\ \binom{i}{2} b_{j-i+2} + a_{j-i} & \text{if } i \ge 2,\ j \ge i \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

where

$$\begin{cases} h_j \ge 0 \quad (j > 0), & 0 \le -h_0 = \sum_{j=1}^{\infty} h_j < \infty \\ a_j \ge 0 \quad (j > 0), & \sum_{j=1}^{\infty} a_j > 0, \quad 0 < -a_0 = \sum_{j \ne 0} a_j < \infty \\ b_j \ge 0 \quad (j \ne 2), & \sum_{j=3}^{\infty} b_j > 0, \quad 0 < -b_2 = \sum_{j \ne 2} b_j < +\infty \end{cases} \tag{1.2}$$

together with $b_0 > 0$ and $b_1 > 0$.

The conditions $b_0 > 0$ and $\sum_{j=3}^{\infty} b_j > 0$ are essential, while condition $b_1 > 0$ is imposed for convenience; all our conclusions hold true with some minor and obvious adjustments if this latter condition is removed.

Definition 1.2. A collision branching process with immigration (henceforth referred to simply as a CBIP) is a continuous-time Markov chain, taking values in $\mathbb{Z}_+$, whose transition function $P(t) = (p_{ij}(t); i, j \in \mathbb{Z}_+)$ satisfies the forward equation

$$P'(t) = P(t)Q \tag{1.3}$$

where Q is a CBI q-matrix defined in (1.1)–(1.2).

In order to avoid discussing some trivial cases, we shall assume that $\mathbb{Z}_+$ is an irreducible class for our q-matrix Q as well as for the corresponding Feller minimal Q-function throughout this paper excepting where we consider the absorbing case. It is easy to see that $\mathbb{Z}_+$ is irreducible if and only if $h_0 \ne 0$, $\sum_{k=0}^{\infty}(a_{2k+1} + b_{2k+1}) > 0$. Therefore, by Theorem 5.1.9 of Anderson (1991), we know that there exists a nonnegative number, $\lambda_Z \ge 0$ say, called the decay parameter of the corresponding process, such that for all $i, j \in \mathbb{Z}_+$,

$$\frac{1}{t} \log p_{ij}(t) \to -\lambda_Z \quad \text{as } t \to +\infty.$$

On the other hand, let

$$\mu_{ij} = \inf\left\{\lambda \ge 0 : \int_0^{\infty} e^{\lambda t} p_{ij}(t)dt = \infty\right\} = \sup\left\{\lambda \ge 0 : \int_0^{\infty} e^{\lambda t} p_{ij}(t)dt < \infty\right\}. \tag{1.4}$$

It is known that irreducibility implies that μ_{ij} is independent of $i, j \in \mathbb{Z}_+$. Denote the common value of μ_{ij} by μ. It is well-known that

$$\lambda_Z = \mu.$$

An elementary but detailed discussion of this theory can be seen in Anderson (1991). For convenience, we briefly repeat these definitions, tailored for our special models, as follows.

Definition 1.3. Let $Q = (q_{ij}; i, j \in \mathbb{Z}_+)$ be a CBI q-matrix such that $\mathbb{Z}_+$ be an irreducible class. Assume that $\mu \ge 0$. A set $(m_i; i \in \mathbb{Z}_+)$ of strictly positive numbers is called a μ-subinvariant measure for Q on $\mathbb{Z}_+$ if

$$\sum_{i \in \mathbb{Z}_+} m_i q_{ij} \le -\mu m_j, \quad j \in \mathbb{Z}_+. \tag{1.5}$$

If equality holds in (1.5), then $(m_i; i \in \mathbb{Z}_+)$ is called a μ-invariant measure for Q on $\mathbb{Z}_+$.

Definition 1.4. Let $P(t) = (p_{ij}(t); i, j \in \mathbb{Z}_+)$ be a CBIP such that $\mathbb{Z}_+$ be an irreducible class. Assume that $\mu \ge 0$. A set $(m_i; i \in \mathbb{Z}_+)$ of strictly positive numbers is called a μ-subinvariant measure for $p_{ij}(t)$ on $\mathbb{Z}_+$ if

$$\sum_{i \in \mathbb{Z}_+} m_i p_{ij}(t) \le e^{-\mu t} m_j, \quad j \in \mathbb{Z}_+. \tag{1.6}$$

If equality holds in (1.6), then $(m_i; i \in \mathbb{Z}_+)$ is called a μ-invariant measure for $p_{ij}(t)$ on $\mathbb{Z}_+$.