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Impulsive-integral inequality and exponential stability for stochastic partial differential equations with delays

Huabin Chen

Department of Mathematics, Nanchang University, Nanchang 330031, Jiangxi, PR China

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A B S T R A C T

In this letter, by establishing an impulsive-integral inequality, some sufficient conditions about the exponential stability in p ($p \geq 2$)-moment of mild solution for impulsive stochastic partial differential equation with delays are obtained. The results in Caraballo and Liu [Caraballo, T. and Liu, K., 1999a. Exponential stability of mild solutions of stochastic partial differential equations with delays. Stoch. Anal. Appl. 17, 743–763] and Luo [Luo, J., 2008b. Fixed points and exponential stability of mild solutions of stochastic partial differential equation with delays. J. Math. Anal. Appl. 342, 753–760] are generalized and improved.

1. Introduction

Stochastic partial differential equations in Hilbert spaces have been studied by some authors and many valuable results on the existence, uniqueness and stability of the solutions have been established; see, for example, Barbu and Bocsan (2002), Caraballo and Liu (1999a,b), Caraballo et al. (2000), Ichikawa (1982, 1983), Liu and Mao (1998), Liu (1998), Taniguchi (1995, 1998, 2007), Liu and Shi (2006), Liu (2006) and references therein. Taniguchi (2007), Wan and Duan (2008) have investigated the exponential stability for stochastic delay differential equations by the energy inequality, respectively; The existence and stability for mild solution of stochastic partial differential equations in Hilbert spaces were considered by applying the comparison theorem in Govindan (2002, 2003); Caraballo and Liu (1999a) have analyzed the exponential stability for mild solution to stochastic partial differential equations with delays by utilizing the well-known Gronwall inequality and they imposed the requirement of the monotone decreasing behaviors of the delays; Taniguchi et al. (2002), have discussed the existence, uniqueness and asymptotic behavior of mild solutions to stochastic partial functional differential equation in Hilbert space by using the semigroup approach; Liu and Truman (2000), Taniguchi (1998) have proved the almost sure exponential stability of mild solution for stochastic partial functional differential equation by using the analytic technique, respectively; Liu and Shi (2006), Liu (2006) have considered the exponential stability for stochastic partial functional differential equations by means of the Razuminkhin-type theorem, respectively.

In the case of differential equations with delays, in particular when we are concerned with the mild solutions of stochastic partial differential equations, the Lyapunov's second method, although it is usually regarded as a powerful tool to study the stability and boundedness, is not suitable to consider such a problem. A difficulty is that mild solutions do not have stochastic differentials, so that one cannot apply the Itô formula to them. Very recently, Burton (2006) has utilized the fixed point theorem to investigate the stability for deterministic systems; and Luo (2007) and Appleby (2008) have applied this valuable method into dealing with the stability for stochastic differential equations. Following the ideas of Burton (2006), Luo (2007) and Appleby (2008), by employing the contraction mapping principle and stochastic integral technique, some sufficient conditions ensuring the exponential stability and asymptotic stability for the mild solution of stochastic partial

E-mail address: chb_00721@126.com.

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differential equations with delays were obtained in Luo (2008a,b), respectively, where the monotone decreasing behaviors of the delays was not comprised.

On the other hand, besides delay effects, impulsive effects likewise exist in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, etc. many interesting results on impulsive effects have been gained in Samoilenko and Perestynk (1995) and references therein. However, to the best of our knowledge, there is no paper which is involved into the existence and exponential stability for the mild solution of impulsive stochastic partial differential equations with delays. Thus, we will make the first attempt to study such problems to close this gap in this letter.

The main difficulty in dealing with exponential stability of mild solutions for impulsive stochastic partial differential equations with delays mainly comes from impulsive effects on the system since the corresponding theory for such problem has not yet been fully developed. Although Sakthivel and Luo (2009a,b) have discussed the asymptotic stability for mild solution of impulsive stochastic partial differential equations by using the fixed point theorem, this very useful method which can be regarded as an excellent tool to derive the exponential stability for mild solution to stochastic partial differential equations with delays in Luo (2008b) may be difficult and even ineffective for the exponential stability of such system with impulses; Besides, it should be pointed out that many methods used frequently fail to consider the exponential stability of mild solution for impulsive stochastic partial differential equations with delays; see, for example, the comparison theorem in Govindan (2002, 2003), the Gronwall inequality in Caraballo and Liu (1999a), the analytic technique in Liu and Truman (2000), Taniguchi (1998) and the semigroup method in Taniguchi et al. (2002). And the methods proposed in Caraballo and Liu (1999a), Caraballo et al. (2000), Ichikawa (1982, 1983), Liu and Mao (1998), Wan and Duan (2008), Liu (1998, 2006) and Taniguchi (2007) are also ineffective in dealing with this problem since mild solutions do not have stochastic differentials. Therefore, techniques and methods for exponential stability of such problem should be developed and explored. This letter presents one such method by establishing an impulsive-integral inequality. Based on the obtained one, we shall give sufficient conditions for the exponential stability in p $(p \geq 2)$-moment of mild solution to impulsive stochastic partial differential equations with delays. The result can generalize and improve the existing works.

2. Preliminaries

Let X and Y be two real, separable Hilbert spaces which are both equipped the norm $\|\cdot\|$, for the sake of convenience. And $L(Y, X)$ denotes the space of bounded linear operators from Y to X. Let $(\Omega, \Im, P)$ be a complete probability space equipped with some filtration $\Im_t$ $(t \geq 0)$ satisfying the usual conditions, i.e., the filtration is right continuous and $\Im_0$ contains all P-null sets.

In this paper, we mainly study the following impulsive stochastic partial differential equations with delays:

$$\begin{cases} \mathrm{d}x(t) = [Ax(t) + f(t, x(t-\delta(t)))]\mathrm{d}t + g(t, x(t-\rho(t)))\mathrm{d}w(t), & t \geq 0, t \neq t_k, \\ \Delta x(t_k) = I_k(x(t_k^-)), & t = t_k, k = 1, 2, \ldots, \\ x_0(\theta) = \varphi \in PC, & \theta \in [-\tau, 0], \quad a.s. \end{cases} \tag{2.1}$$

where φ is $\Im_0$-measurable and the functions: $\delta,\ \rho : [0, +\infty) \to [0, \tau]$ $(\tau > 0)$ are continuous. Let $PC \equiv PC([-\tau, 0]; X)$ be the space of all almost surely bounded, $\Im_0$-measurable and continuous functions everywhere except for a infinite number of point s at which $\xi(s)$ and the left limit $\xi(s)$ exist and $\xi(s^+) = \xi(s)$ from $[-\tau, 0]$ into X and as usual, equipped with the supremum norm $\|\varphi\|_0 = \sup_{\theta\in[-\tau,0]} \|\varphi(\theta)\|$; A is the infinitesimal generator of a semigroup of bounded linear operators $S(t)$ $(t \geq 0)$ in X, and the readers can refer to Pazy (1983); Moreover, the fixed moments of time t_k satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$, and $\lim_{k\to+\infty} t_k = \infty$; $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, $k = 1, 2, \ldots$, respectively; $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ denotes the jump in the state x at time t_k with $I_k(\cdot) : X \to X$ $(k = 1, 2, \ldots)$ determining the size of the jump; $f : [0, +\infty) \times X \to X$ and $g : [0, +\infty) \times X \to L_2^0(Y, X)$ are all suitable Borel measurable mappings, where the space $L_2^0(Y, X)$ is introduced in detail as follows.

Let $\beta_n(t)$ $(n = 1, 2, \ldots)$ be a sequence of real-valued one dimensional standard Brownian motions mutually independent over $(\Omega, \Im, P)$. Set $w(t) = \sum_{n=1}^{+\infty} \sqrt{\lambda_n}\beta_n(t)e_n$ $(t \geq 0)$, where $\lambda_n \geq 0$ $(n = 1, 2, \ldots)$ are nonnegative real numbers and $\{e_n\}$ $(n = 1, 2, \ldots)$ is a complete orthonormal basis in Y. Let $Q \in L(Y, Y)$ be an operator defined by $Qe_n = \lambda_n e_n$ with a finite trace $\mathrm{tr}\, Q = \sum_{n=1}^{+\infty} \lambda_n < +\infty$. Then, the above Y-valued stochastic process $w(t)$ is called a Q-Wiener process.

Definition 2.1. Let $\sigma \in L(Y, X)$ and defined

$$\|\sigma\|_{L_2^0}^2 := \mathrm{tr}(\sigma Q \sigma^*) = \left\{\sum_{n=1}^{+\infty} \|\sqrt{\lambda_n}\sigma e_n\|^2\right\}.$$

If $\|\sigma\|_{L_2^0}^2 < +\infty$, then σ is called a Q-Hilbert–Schmidt operator and let $L_2^0(Y, X)$ denote the space of all Q-Hilbert–Schmidt operators $\sigma : Y \to X$.

Now, for the definition of a X-valued stochastic integral of an $L_2^0(Y, X)$-valued, $\Im_t$-adapted predictable process $h(t)$ with respect to the Q-Wiener process $w(t)$, we can refer to Prato and Zabczyk (1992).