Statistical Methodology 32 (2016) 63–76

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Testing variability orderings by using Gini's mean differences

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ARTICLE INFO

Article history:
Received 24 April 2015
Received in revised form
5 February 2016
Accepted 8 March 2016
Available online 18 March 2016

Keywords:
Testing variability
Gini's mean difference estimators
Stochastic orders in variability
Log-returns distributions

ABSTRACT

In this paper, we derive a measure of discrepancy based on the Gini's mean difference to test the null hypothesis that two random variables, which are ordered in a variability-type stochastic order, are equally dispersive versus the alternative that one strictly dominates the other. We describe the test, evaluate its performance under a variety of situations and illustrate the procedure with an example using log returns of real data.

1. Introduction

Stochastic orders are important tools for making ordinal comparisons among probability distributions (or random variables) in terms of some characteristics, such as location or variability (see [20,29] for an overview of the topic). Once these ordinal comparisons have been established between two distributions, they can be of great use in applications such as finance, actuarial, reliability and many other fields. In practice, in order to determine whether a stochastic ordering holds between two distributions of data, diverse testing procedures have been developed. In this paper, we study and illustrate a testing method for various stochastic variability orderings which is particularly useful when dealing with heavy-tailed distributions.

Stochastic orders are often in agreement with cardinal comparisons made among specific measures of the characteristic in question. Thus, for example, most of the classical variability orders, including the dispersive order, the convex order, the dilation order and the excess wealth order (see Section 3

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http://dx.doi.org/10.1016/j.stamet.2016.03.001
1572-3127/

in [29]) and others more recent, such as the tail-dispersive order introduced by Sordo, Souza and Suárez-Llorens [34] agree with the comparison of variances and Gini's mean differences. This means that if $X \leq_{(*)} Y$, where $\leq_{(*)}$ denotes some of the above orders, then $Var(X) \leq Var(Y)$ and $GMD(X) \leq GMD(Y)$, where $GMD(X)$ denotes the Gini's difference of X, defined by

$$GMD(X) = E\left|X_1 - X_2\right|, \tag{1}$$

with X_1 and X_2 being two independent copies of X. In this paper, we use the consistency of the Gini's mean difference with the above orders to construct suitable tests of equality of two ordered random variables.

The Gini's mean difference is a variability measure that shares many properties with the variance (see [39]). Like the variance (which can be defined as $Var(X) = \frac{1}{2}E(X_1 - X_2)^2$, where X_1 and X_2 are two independent copies of X) the Gini's mean difference can be defined without reference to a specific location measure. However, while the variance is superior to the Gini's mean difference for distributions that are nearly normal, the Gini's mean difference can be more informative for distributions that depart from the normal. In finance, insurance and other fields, the deviation from normality is often caused by heavy tails in distributions. In many such situations, the variance does not exist since it involves the second moment (that is the case, for example, of a Pareto distribution with shape parameter $\alpha \leq 2$). In these situations, the Gini's mean difference becomes specially useful since it only requires the existence of the mean. In fact, the Gini's mean difference plays an important role as a measure of right tail risk in insurance (see [38]).

We consider in this paper two random variables X and Y such that $X \leq_{(*)} Y$, where $\leq_{(*)}$ denotes the dispersive order, the excess wealth order, the convex order, the tail-dispersive order or the dilation order. Our purpose is to illustrate the problem of testing the null hypothesis $H_0 : X =_{(*)} Y$ against the alternative $H_1 : X \leq_{(*)} Y$ and $X \neq_{(*)} Y$, by using a statistical test based on the empirical Gini's mean differences associated to two independent random samples of X and Y. In contrast to other specific tests proposed in the literature for each of these orders (see for example [1,18] for the dispersive order, Belzunce et al. [6] for the excess wealth order, Berrendero and Cárcamo [10,11] for the convex order and Belzunce et al. [3,7] for the dilation order), a test based on Gini's mean differences presents the advantage of being applicable to all the orders mentioned above. This approach, however, also presents some difficulties arising from the estimation process. In this work, we make a revision of the literature on this issue, describe the test, evaluate its performance under a variety of situations and illustrate the procedure with an example using real data. To that end, the paper is organized as follows. In Section 2, we recall the definitions of the variability stochastic orders under consideration and give the results that justify the statistical test. In Section 3, we review four estimators of the Gini's mean difference given, respectively, by Glasser [14], Nygard and Sandström [22] and Zenga, Polisicchio and Greselin [41]. In Section 4, we discuss the performance of different tests for the above variability orders based on these estimators by comparing their power functions under particular distributions. Section 5 contains a numerical example applied to log returns distributions and Section 6 some further remarks.

2. Variability orders and Gini's mean difference

Given a random variable X with distribution function F, we denote by $F^{-1}(p) \equiv \inf\{x : F(x) \geq p\}$, $p \in (0, 1)$, the corresponding quantile function. For any real number a we use the notation $a^+ = \max\{a, 0\}$. First, we define the variability stochastic orders considered in this paper.

Definition 2.1. Let X and Y be two random variables with respective distribution functions F and G. Then, we say that:

1. X is smaller than Y in the dispersive order, denoted by $X \leq_{disp} Y$, if

$$F^{-1}(p) - F^{-1}(q) \leq G^{-1}(p) - G^{-1}(q) \quad \text{for all } 0 < q < p < 1.$$

2. X is smaller than Y in the excess wealth order, denoted by $X \leq_{ew} Y$, if

$$E\left[\left(X - F^{-1}(p)\right)^+\right] \leq E\left[\left(Y - G^{-1}(p)\right)^+\right], \quad \text{for all } p \in (0, 1).$$