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Estimation and prediction for a progressively censored generalized inverted exponential distribution

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HIGHLIGHTS

- Bayesian and classical estimates are derived for the unknown parameters of a generalized inverted exponential distribution based on progressively Type-II censored data.
- Prediction of future failures are discussed using classical and Bayesian approaches.
- A simulation study is conducted to assess the behavior of proposed methods and two real data sets are analyzed for illustrative purposes.

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ABSTRACT

In this paper, we consider generalized inverted exponential distribution which is capable of modeling various shapes of failure rates and aging criteria. The purpose of this paper is two fold. Based on progressive type-II censored data, first we consider the problem of estimation of parameters under classical and Bayesian approaches. In this regard, we obtain maximum likelihood estimates, and Bayes estimates under squared error loss function. We also compute 95% asymptotic confidence interval and highest posterior density interval estimates under the respective approaches. Second, we consider the problem of prediction of future observations using maximum likelihood predictor, best unbiased predictor, conditional median predictor and Bayes predictor. The associated predictive interval estimates for the censored observations are computed as well. Finally, we analyze two real data sets and conduct a

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Monte Carlo simulation study to compare the performance of the various proposed estimators and predictors.

1. Introduction

The one-parameter exponential distribution is the simplest and most widely used lifetime model in life testing and reliability analysis. In spite of its popularity, this distribution has certain restrictions like constant hazard rate. To make its applicability more flexible, a large amount of work has been done via exponentiating the distribution function (see Gupta and Kundu [16]). Another modification to this distribution has been done by using its inverted version, known as the inverted exponential (IE) distribution. Lin et al. [24] studied one-parameter IE distribution and obtained maximum likelihood estimator, confidence limits, and UMVUE for the parameter and the reliability function based on complete samples. They also compared this model with inverted Gaussian and log-normal distributions based on maintenance data set, and observed that it provides a better fit than these two distributions. Further, Dey [10] obtained Bayes estimators of the parameter and risk functions under different loss functions. Abouammoh and Alshingiti [1] introduced a shape parameter to the IE distribution to obtain generalized inverted exponential (GIE) distribution. The probability density function (PDF) and cumulative distribution function (CDF) of GIE(γ, λ) distribution are respectively given by

$$f(x;\gamma,\lambda) = \frac{\gamma\lambda}{x^2} e^{-\frac{\lambda}{x}} \left(1 - e^{-\frac{\lambda}{x}}\right)^{\gamma-1}, \quad x > 0,\ \gamma > 0,\ \lambda > 0, \tag{1}$$

$$F(x;\gamma,\lambda) = 1 - (1 - e^{-\frac{\lambda}{x}})^{\gamma}. \tag{2}$$

Notice that here γ is the shape parameter and λ is the scale parameter of GIE distribution, and moreover correspond to $\gamma = 1$ GIE distribution reduces to IE distribution. It is observed that the hazard function of GIE distribution can be increasing or decreasing but not constant, depending on the shape parameter. It is also observed that in many situations this distribution may provide a better fit than gamma, Weibull, and generalized exponential distributions (see Abouammoh and Alshingiti [1]). For recent contributions to this distribution, one may refer to Krishna and Kumar [19], Dey and Dey [12], Dey and Pradhan [14], Dey and Dey [11], Dey et al. [13], Singh et al. [30], Singh et al. [33], and Dube et al. [15].

In recent past, progressive type-II censoring has received much attention in the literature of life testing and reliability analysis. The main advantage of this censoring over the traditional type-II censoring is that the removal of live units at intermediate stages is allowed under this censoring, whereas, under type-II censoring live units can only be removed at the time of the termination of experiment. Lifetime data under this censoring can be collected in the following way. Suppose that a sample of n independent and identical units is put on a life test experiment. Further assume that life times of the units follow PDF $f(x;\theta)$ and CDF $F(x;\theta)$, here θ is a vector of unknown parameters of the distribution. Now as the experiment will start the units on the test will start failing, let us suppose that first failure occurs at a random time $X_{(1)}$. Then under this censoring at the time $X_{(1)}$, R_1 number of live units are removed from the remaining $n-1$ units in the experiment. In a similar way, when a second failure occurs at a random time $X_{(2)}$, R_2 number of live units are randomly removed from the remaining $n-2-R_1$ units, and so on, and at the time of mth failure $X_{(m)}$, all the remaining $n-m-\sum_{i=1}^{m-1} R_i$ number of units are removed, and the experiment is terminated. Here, the censoring scheme $R = (R_1, R_2, \ldots, R_m)$ is prefixed prior to the commencement of the experiment so that $\sum_{i=1}^{m} R_i = n - m$. It is to be noted that corresponding to $R_1 = R_2 = \cdots = R_{m-1} = 0$ and $R_m = n - m$, this censoring reduces to the traditional type-II censoring. Further, corresponding to $m = n$ and $R_i = 0, i = 1, 2, \ldots, n$, it reduces to complete sample having no censoring. Therefore,