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Some new results on the LQE ordering

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ABSTRACT

Ebrahimi and Pellerey (1995) and Ebrahimi (1996) proposed the residual entropy. Recently, Sunoj and Sankaran (2012) obtained a quantile version of the residual entropy, the residual quantile entropy (RQE). Based on the RQE function, they defined a new stochastic order, the less quantile entropy (LQE) order, and studied some properties of this order. In this paper, we focus on further properties of this new order. Some characterizations of the LQE order are investigated, closure and reversed closure properties are obtained, meanwhile, some illustrative examples are shown. As applications of a main result, the preservation of the LQE order in several stochastic models is discussed. We give the closure and reversed closure properties of the LQE order for coherent systems with dependent and identically distributed components, and also consider a potential application to insurance of this order.

1. Introduction

In recent years, there has been a great interest in the measurement of uncertainty of probability distribution. Let X be a nonnegative absolutely continuous random variable representing the lifetime of a component or a system. The Shannon entropy of X is defined by

$$H_X = H(X) = -\mathrm{E}[\ln f_X(X)] = -\int_0^{+\infty} f_X(x)\ln f_X(x)\mathrm{d}x. \tag{1.1}$$

In continuous case, H_X is also referred to as the Shannon differential entropy. The properties of H_X have been thoroughly investigated due to the classical contributions by Shannon [39] and Wiener [44]. The Shannon entropy plays a central role in the field of information theory and has a wide range of

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applications in many fields. In the literature some dynamic generalizations of H_X have been proposed (see, for instance, Khinchin [19], Taneja [42], Gupta and Nanda [17], Nanda and Paul [25], Asadi and Zohrevand [2], Di Crescenzo and Longobardi [9], Kumar and Taneja [23], Khorashadizadeh et al. [20] and the references therein).

Ruiz and Navarro [35] defined the inactivity time of X at time t as the random variable $X_{(t)} = [t - X \,|X \le t\,]$ for all $t \ge 0$. The distribution function and density function of $X_{(t)}$ are given, respectively, by

$$F_{X_{(t)}}(x) = \frac{F_X(t-x)}{F_X(t)}, \qquad f_{X_{(t)}}(x) = \frac{f_X(t-x)}{F_X(t)}, \quad 0 \le x \le t.$$

Di Crescenzo and Longobardi [7] introduced the past entropy as

$$\overline{H}_X(t) = -\int_0^t \frac{f_X(x)}{F_X(t)} \ln\left[\frac{f_X(x)}{F_X(t)}\right] \mathrm{d}x, \quad \text{for all } t \ge 0. \tag{1.2}$$

That is, $\overline{H}_X(t) = H_{X_{(t)}} = H(X_{(t)})$. They showed that the past entropy $\overline{H}_X(t)$ can be written as

$$\overline{H}_X(t) = \ln F_X(t) - \frac{1}{F_X(t)} \int_0^t f_X(x) \ln f_X(x) \mathrm{d}x. \tag{1.3}$$

Ebrahimi and Pellerey [13] and Ebrahimi [10] proposed the residual entropy as a useful dynamic measure of uncertainty. The residual life of X is defined by $X_t = [X - t\,|X > t\,]$ for all $t \ge 0$. Then X_t has survival and density functions $\overline{F}_{X_t}(x)$ and $f_{X_t}(x)$ given by, respectively,

$$\overline{F}_{X_t}(x) = \frac{\overline{F}_X(x+t)}{\overline{F}_X(t)}, \qquad f_{X_t}(x) = \frac{f_X(x+t)}{\overline{F}_X(t)}, \quad x \ge 0. \tag{1.4}$$

The residual entropy of X at time t is defined as the differential entropy of X_t. Formally, for all $t \ge 0$, the residual entropy of X is given by $H_X(t) = H_{X_t} = H(X_t) = -\mathrm{E}[\ln f_{X_t}(X_t)]$. They showed that

$$H_X(t) = -\int_t^{+\infty} \frac{f_X(x)}{\overline{F}_X(t)} \ln \frac{f_X(x)}{\overline{F}_X(t)} \mathrm{d}x. \tag{1.5}$$

Given that a component survived up to time t, then $H_X(t) = H_{X_t}$ measures the uncertainty of the residual life X_t. Belzunce et al. [6] showed that for an absolutely continuous random variable with an increasing residual entropy, the residual entropy function uniquely determines the distribution function. Many other authors did their research works about the residual entropy, various results concerning the residual entropy $H_X(t)$ have been obtained by Ebrahimi and Kirmani [11,12], Asadi and Ebrahimi [1], Di Crescenzo and Longobardi [7,8], Nanda and Paul [26,27], Kundu and Nanda [24], Navarro et al. [28] and the references therein.

Recently, Sunoj and Sankaran [41] obtained a quantile version of the residual entropy, the residual quantile entropy (RQE). They showed that the residual quantile entropy is efficient and convenient and is an equivalent alternative to the residual entropy in describing aging properties of life distributions and in making stochastic comparisons. On the basis of RQE function, they defined two new nonparametric classes of life distributions, decreasing (increasing) residual quantile entropy (DRQE (IRQE)) class, and they defined a new stochastic order, the less (residual) quantile entropy (LQE) order, to compare the uncertainties of residual lives of two random lives X and Y at the age points with equal survival probabilities.

The following lemma taken from Barlow and Proschan [3] plays an important role in the proofs of the paper.

Lemma 1.1. *Let W be a measure on the interval (a, b), not necessarily nonnegative, where $-\infty \le a < b \le +\infty$. Let h be a nonnegative function defined on (a, b). If $\int_t^b \mathrm{d}W(x) \ge 0$ for all $t \in (a, b)$ and if h is increasing, then $\int_a^b h(x)\mathrm{d}W(x) \ge 0$.*