Statistics and Probability Letters 83 (2013) 624–630

Contents lists available at SciVerse ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Optimal blocking and semifoldover plans for 2^{n-p} designs

Po Yang *

Department of Statistics, University of Manitoba, Winnipeg, MB R3T 2N2, Canada

ARTICLE INFO

Article history:
Received 9 August 2012
Received in revised form 29 October 2012
Accepted 29 October 2012
Available online 12 November 2012

Keywords:
Foldover
Semifoldover
Blocked design
Aliasing
Resolution

ABSTRACT

This article studies the semifoldover designs when initial designs are blocked regular designs. The properties and the structures of semifoldover blocked designs are investigated. Optimal blocking and semifoldover plans are tabulated for 16 and 32 run designs.

1. Introduction

Two-level fractional factorial designs are commonly used in experiments with several factors. A two-level regular fractional factorial design 2^{n-p} is generated by p *generators*, where n is the number of factors of the design and $n - p$ is the number of *independent treatment factors* or *basic factors*. The *defining relation*, which provide alias relations of the design, is generated by the p generators. Each alias relation in the defining relation is called a *word* of the design. For example, consider a 2^{6-2} design generated by two generators $5 = 123$ and $6 = 124$. The basic factors are 1, 2, 3, and 4 and the defining relation of the design is $I = 1235 = 1246 = 3456$. The words 1235, 1246, and 3456 provide the alias structure of the design. For instance, 1235 implies that the two-factor interactions 12 and 35 are aliased or confounded and, therefore, cannot be estimated.

In order to break aliasing, adding another fraction to an initial fractional factorial design may be necessary. Foldover is a technique used to select the follow-up runs of the initial design by revising the signs of one or more of its columns. For an initial 2^{n-p} design, we can get 2^{n-p} new runs by revising the signs of r columns of the design. Following Li and Lin (2003), we call a collection of the r columns a *foldover plan*. The new runs form a new fraction, called a *foldover fraction*. The combination of the initial design and the new fraction is called a *foldover design*. Box et al. (2005) and Montgomery and Runger (1996) study the foldover plan of reversing the signs of one column of the initial design to de-alias the factor of interest from other factors. Li and Mee (2002) point out that revising signs of all factors may not be the best choice for resolution III designs. They discussed better foldover plans for resolution III designs. Li and Lin (2003) develop optimal foldover plans for regular designs with 16 and 32 runs using the minimum aberration criterion of the foldover designs.

It is well-known that the foldover fraction includes all the words of the initial design, but some of them change signs. Assume that $W = W_1 \cup W_2$ contains all the words of the initial design, where W_1 includes the words which change signs and W_2 contains the words which do not change signs after a foldover plan. In particular, we assume that the identity $I \in W_2$. It is well-known that the foldover design includes only the words in W_2. Since the new runs are usually performed at a different time, it is important that we consider a block factor B. Following Ai et al. (2010), the block factor is called the *implicit blocking*

* Tel.: +1 204 480 1821.
E-mail addresses: yangp34@cc.umanitoba.ca, Po.Yang@ad.umanitoba.ca.

0167-7152/$ – see front matter
doi:10.1016/j.spl.2012.10.027

factor. Let B be 1 in the initial design and -1 in the new fraction. Then, the foldover design with consideration of B includes the words in $W_2 \cup BW_1$. Ye and Li (2003) show that the inclusion of B does not change the comparison of the foldover designs with respect to their aberration.

A semifoldover design is obtained by adding only half of the new runs to the initial design. Barnett et al. (1997) study a semifoldover design through a practical example and pointed out that a semifoldover design can work as well as the corresponding foldover design in terms of de-aliasing effects. A semifoldover design can be obtained by adding the new runs which satisfy $X = 1$, thus, only half of the new runs, or the new runs which such that $X = -1$, to the initial design, here, X is a main factor or an interaction. Since the semifoldover design, obtained by adding the new runs which satisfy $X = 1$, has the same alias structure as the one obtained by adding the new runs which satisfy $X = -1$ (see Balakrishnan and Yang, 2009), in this article, we refer to the first one as the semifoldover design and say that it is obtained by *folding over on r factors* and *subsetting on X*. Following Mee and Peralta's (see Mee and Peralta, 2000) notation, the *semifoldover plan* is denoted by $fo = r$ factors and $ss = X$. Here $ss = X$ is called a *subsetting plan*.

Mee and Peralta (2000) and John (2000) show theoretically that a semifoldover design can estimate as many main effects and two-factor interactions as the corresponding foldover design. Huang et al. (2008) determine optimal semifoldover plans in terms of de-aliasing the maximum number of main factors and two-factor interactions criteria. Balakrishnan and Yang (2009) and Edwards (2011) study semifoldover nonregular designs and Edwards (2011) provides optimal semifoldover orthogonal designs using the criterion of minimum dependent sets. Yang and Li (2012) consider semifoldover regular designs with consideration of B and show that the inclusion of B may change the choice of optimal semifoldover designs.

Given an initial design, there are many foldover plans. Li and Lin (2003) define *core foldover plans* as the plans which contain only generators and they show that any foldover plan is equivalent to a core foldover plan. Similarly, given an initial design, there are many subsetting plans. A *core subsetting plan* is defined as the subsetting plan which contains only basic treatment factors. Yang and Li (2012) show that any subsetting plan is equivalent to a core subsetting plan. These results are very useful when searching optimal semifoldover designs since we do not need to consider all possible semifoldover plans and, therefore, significantly save searching time.

Two combined designs are considered in a semifoldover design. The combined design 1, denoted by F_1, includes the initial runs which satisfy $X = 1$ and the half new runs. The combined design 2, denoted by F_2, contains the initial runs which satisfy $X = -1$ and the half new runs. By Balakrishnan and Yang (2009), F_1 contains the words in $U_1 = W_2 \cup XW_2$ and F_2 contains the words in $U_2 = W_2 \cup XW_1$. When B is included, Yang and Li (2012) provide that F_1 includes the words in $U_1 \cup BW_1 \cup BXW_1$ and F_2 contains the words in $U_2 \cup BW_1 \cup BXW_2$.

Blocking is a technique commonly used in experimental designs and a method to reduce experimental error variation. Li and Jacroux (2007) consider foldover blocked designs when initial designs are blocked designs. They provide optimal foldover plans using two aberration criteria. Ai et al. (2010) investigate structures and properties of foldover blocked designs and provide optimal blocking and foldover plans using different criteria. In this article, we study semifoldover designs when initial designs are blocked designs. Section 2 introduces the structure of a semifoldover blocked design. We show that given a resolution IV design and a blocking plan, a semifoldover blocked design can estimate as many two-factor interactions as the corresponding foldover blocked design. We also prove that two semifoldover blocked designs, obtained by two different foldover plans, respectively, and subsetting on the same treatment effect, can de-alias the same effects. The two foldover plans contain the same treatment factors but different block factors. In Section 3, a catalog of optimal blocking and semifoldover plans for designs with 16 and 32 runs in Chen et al. (1993) is provided. In particular, the criterion used for finding optimal plans is the maximum number of main factors and two-factor interactions that can be de-aliased.

Throughout this article, we assume that: (1) interactions of three or more treatment factors are negligible, (2) any main treatment factor is not aliased with block factors in the initial blocked design, and (3) following Wu and Hamada (2000), interactions between block factors and treatment factors are negligible.

2. Structures and properties of semifoldover blocked designs

Consider a blocked 2^{n-p} design with k block factors $b_1 = v_1, b_2 = v_2, \ldots, b_k = v_k$, where $v_1, v_2, \ldots, v_k$ are interactions of the $n - p$ independent treatment factors. The k block factors can be considered as k block generators. This blocked design is denoted by a $(2^{n-p} : 2^k)$ design. The defining relation of the blocked design includes two types of words: one type of the words contains only treatment factors and the other contains both treatment factors and block factors. So, in the rest of the article, we assume that $W_1 = W_{1,t} \cup W_{1,b}$ and $W_2 = W_{2,t} \cup W_{2,b}$, where W_1 and W_2 are defined in Section 1, $W_{1,t}$ and $W_{2,t}$ contain the words which include only treatment factors, and $W_{1,b}$ and $W_{2,b}$ contain the words which include both treatment factors and block factors in the initial design.

Therefore, from the discussion in Section 1, we can obtain that for a semifoldover blocked design, without consideration of B, F_1 contains the words in $U_1 = W_{2,t} \cup W_{2,b} \cup XW_{2,t} \cup XW_{2,b}$ and F_2 includes the words in $U_2 = W_{2,t} \cup W_{2,b} \cup XW_{1,t} \cup XW_{1,b}$. With consideration of B, F_1 contains the words in $U_1 \cup BW_{1,t} \cup BW_{1,b} \cup BXW_{1,t} \cup BXW_{1,b}$ and F_2 includes the words in $U_2 \cup BW_{1,t} \cup BW_{1,b} \cup BXW_{2,t} \cup BXW_{2,b}$.

Given an initial blocked design, a subsetting plan may contain treatment factors and/or block factors. Note that block factors are generated by independent treatment factors. By Yang and Li (2012), any subsetting plan is equivalent to a core subsetting plan which contains only independent treatment factors. Therefore, in the rest of the article, we do not consider the subsetting plans which contain block factors. Similarly, given an initial blocked design, a foldover plan may also contain