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A class of distributions with the linear mean residual quantile function and it's generalizations

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ABSTRACT

In the present paper, we introduce and study a class of distributions that has the linear mean residual quantile function. Various distributional properties and reliability characteristics of the class are studied. Some characterizations of the class of distributions are presented. We then present generalizations of this class of distributions using the relationship between various quantile based reliability measures. The method of L-moments is employed to estimate parameters of the class of distributions. Finally, we apply the proposed class of distributions to a real data set.

1. Introduction

In modeling and analysis of statistical data with probability distributions, there are two equivalent approaches, one is through the distribution function and the other is through the quantile function defined by

$$Q(u) = F^{-1}(u) = \inf\{x : F(x) \geq u\}, \quad 0 \leq u \leq 1 \tag{1.1}$$

where $F(x)$ is the distribution function of random variable X. Even though both convey the same information about the distribution with different interpretations, the concepts and methodologies based on distribution functions are habitually employed in most forms of statistical studies. However, quantile functions have several properties that are not shared by distributions, which make it more convenient for analysis. For example, the sum of two quantile functions is again a quantile function.

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For more properties and applications of quantile functions one could refer to Parzen [25], Gilchrist [3], Nair et al. [19] and Nair and Sankaran [17].

In reliability studies, the distribution function $F(x)$, the associated survival function $\bar{F}(x) = 1-F(x)$ and the probability density function $f(x)$ along with various other characteristics such as hazard rate, mean, percentiles and higher moments of residual life etc, are used for understanding the generating mechanism of the lifetime data and to distinguish between various models through aging properties. Among those various concepts, the mean residual function is a well known measure, which has been widely used in the fields of reliability, statistics, survival analysis and insurance. For a non-negative random variable x, the mean residual life function is defined as

$$m(x) = E(X - x|X > x) = \frac{1}{\bar{F}(x)} \int_x^\infty \bar{F}(t)dt.$$

It is interpreted as the expected remaining lifetime of a unit given survival up to time x. Muth [14] and Guess and Proschan [4] discuss the basic results and various applications of the mean residual life function. In an alternative approach Nair and Sankaran [16] view the mean residual life function as the expectation of the conditional distribution of residual life given age, arising from the joint distribution of age and residual life in renewal theory. Gupta and Kirmani [6] have provided characterizations of lifetime distributions using the mean residual life function. The class of distributions with linear mean residual life has been studied by Hall and Wellner [7,8] and Gupta and Bradley [5]. This class of distributions with linear mean residual life contains Pareto, an exponential and rescaled beta distribution. Oakes and Dasu [24] and Korwar [11] have developed characterizations for the class of linear mean residual life distributions. Chen and Cheng [1] and Nanda et al. [22] studied proportional mean residual life model for the analysis of survival data.

Recently, Nair and Sankaran [15] have introduced the basic concepts in reliability theory in terms of quantile functions. Let X be a non-negative random variable with distribution function $F(x)$, satisfying $F(0) = 0$, $F(x)$ is continuous and strictly increasing. Nair and Sankaran [15] defined the mean residual quantile function which is given by

$$M(u) = \frac{1}{1-u} \int_u^1 (Q(p) - Q(u))dp. \tag{1.2}$$

We can interpret $M(u)$ as the mean remaining life of a unit beyond the $100(1-u)\%$ of the distribution. In the present study, we consider a class of distributions with the linear mean residual quantile function given by

$$M(u) = cu + \mu, \quad \mu > 0, \ -\mu \le c < \mu, \ 0 \le u \le 1. \tag{1.3}$$

This class includes various well known distributions.

The rest of the article is organized as follows. In Section 2 we present a class of distributions with the linear mean residual quantile function and study its basic properties. Distributional properties like measures of location and scale, L moments, order statistics etc. are given in Section 3. In Section 4 we present approximation of some well known distribution with the proposed class of distribution. In Section 5 we present various reliability characteristics of the class of distributions and provide four characterization theorems. In Section 6 we present general classes of distributions in which the linear mean residual quantile function is a member. Section 7 focuses on the inference procedures for the class of distributions with the linear mean residual quantile function. Finally we provide an application of this class of distributions in a real life situation.

2. A class of distributions

As mentioned in Section 1, we consider a class of distributions with

$$M(u) = cu + \mu.$$