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Global attracting set and stability of stochastic neutral partial functional differential equations with impulses

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ABSTRACT

In this paper, a class of stochastic neutral partial functional differential equations with impulses is investigated. To this end, we first establish a new impulsive-integral inequality, which improve the inequality established by Chen [Chen, H.B., 2010. Impulsive-integral inequality and exponential stability for stochastic partial differential equation with delays. Statist. Probab. Lett. 80, 50–56]. By using the new inequality, we obtain the global attracting set of stochastic neutral partial functional differential equations with impulses. Especially, the sufficient conditions ensuring the exponential p-stability of the mild solution of the considered equations are obtained. Our results can generalize and improve the existing works. An example is given to demonstrate the main results.

1. Introduction

Recently, there has been increasing interest in the study of the existence, uniqueness and stability of mild solutions of stochastic partial functional differential equations due to their range of applications in various sciences such as physics, mechanical, engineering, control theory and economics, and many significant results have been obtained, see, for example, Caraballo (1990), Caraballo and Liu (1999), Govindan (1999), Liu (1998, 2006, 2009), Luo (2008a,b), Taniguchi et al. (2002) and Wan and Duan (2008). However, many dynamical system not only depend on present and past states but also involve derivatives with delays. Neutral functional differential equations are often used to describe such systems. In literature, there are only a few works on the stability of mild solutions to stochastic neutral partial functional differential equations, see, for example, Caraballo et al. (2007), Cui et al. (2011), Govindan (2005), Liu and Xia (1999), Luo (2009) and Sakthivel et al. (2010). One of the reasons is that the mild solutions do not have stochastic differentials, so Itô's formula fails to deal with this problem.

In addition, impulsive phenomena can be found in a wide variety of evolutionary processes, for example, medicine and biology, economics, mechanics, electronics and telecommunications, etc., in which many sudden and abrupt changes occur instantaneously, in the form of impulses. Many interesting results have been obtained, see for example, Anguraj and Vinodkumar (2010a,b), Chen (2010) and Sakthivel and Luo (2009a,b). When we consider the exponential stability of

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mild solutions for impulsive stochastic partial differential equations with delays, the main difficulty mainly comes from impulsive effects on the system since the corresponding theory for such a problem has not yet been fully developed. Although Anguraj and Vinodkumar (2010a) deal with the existence, uniqueness and asymptotic stability of impulsive stochastic neutral partial functional differential equations with infinite delays by using Picard's iterative method and stochastic integral technique; Sakthivel and Luo (2009a,b) have discussed the asymptotic stability for mild solutions of impulsive stochastic partial differential equations by using the fixed point theorem which can be regarded as an excellent tool to derive the exponential stability for mild solutions to stochastic partial differential equations with delays in Luo (2008a, 2009). This very useful method may be difficult and even ineffective for the exponential stability of such system with impulses. In addition, some other methods used in Caraballo and Liu (1999) and Wan and Duan (2008) et al. are also ineffective for this problem since mild solutions do not have stochastic differentials. For the above reasons, Chen (2010) established an impulsive-integral inequality to investigate the exponential stability of a stochastic partial differential equation with delays and impulses, but it is not effective for the neutral type. Besides the stability, the global attracting set has been widely studied for various systems, see, for example, Bernfeld et al. (2003), Caraballo et al. (2007), Liao et al. (2008), Xu (2003) and Xu and Zhao (2002). Caraballo et al. (2007) deal with the almost sure exponential stability and ultimate boundedness of the solutions to a class of neutral stochastic semilinear partial delay differential equations. However, to the best of our knowledge, there are no results on the exponential p-stability and global attracting set of impulsive stochastic neutral partial functional differential equations. On the basis of this, this article is devoted to the discussion of this problem.

Motivated by the above discussion and methods, we first establish a new impulsive-integral inequality, which improves the inequality established by Chen (2010), such that it is effective for neutral systems. Next, by using the stochastic analysis techniques, the properties of operator semigroup, and combining the new impulsive-integral inequality, we investigate the global attracting set and exponential p-stability of stochastic neutral partial functional differential equations with impulses, and obtain the global attracting set and sufficient conditions for exponential p-stability of the considered system. Our results can generalize and improve the existing works.

2. Model description and preliminaries

Throughout this paper, unless otherwise specified, we use the following notations. Let H, K be real separable Hilbert spaces and $\mathcal{L}(K, H)$ be the space of bounded linear operators mapping K into H. For convenience, we shall use the same notations $\|\cdot\|$ to denote the norms in H, K and $\mathcal{L}(K, H)$ without any confusion. Let $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathscr{F}\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathscr{F}_0$ contains all P-null sets). Let $\{\omega(t): t \geq 0\}$ denote a K-valued $\{\mathscr{F}\}_{t\geq 0}$-Wiener process defined on $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, P)$ with covariance operator Q, i.e.,

$$\mathbb{E}\langle \omega(t), x\rangle_K \langle \omega(s), y\rangle_K = (t \wedge s)\langle Qx, y\rangle_K \quad \text{for all } x, y \in K,$$

where Q is a positive self-adjoint, trace class operator on K, $\langle \cdot, \cdot \rangle_K$ denotes the inner product of K, $\mathbb{E}$ denotes the mathematical expectation. In particular, we call such $\omega(t)$, $t \geq 0$, a K-valued Q-Wiener process with respect to $\{\mathscr{F}\}_{t\geq 0}$.

In order to define stochastic integrals with respect to the Q-Wiener process $\omega(t)$, we introduce the subspace $K_0 = Q^{1/2}(K)$ of K which, endowed with the inner product $\langle u, v\rangle_{K_0} = \langle Q^{-1/2}u, Q^{-1/2}v\rangle_K$, is a Hilbert space. We assume that there exists a complete orthonormal system $\{e_i\}_{i\geq 1}$ in K, a bounded sequence of nonnegative real numbers λ_i such that $Qe_i = \lambda_i e_i$, $i = 1, 2, \ldots$, and a sequence $\{\beta_i(t)\}_{i\geq 1}$ of independent Brownian motions such that

$$\langle \omega(t), e\rangle = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\langle e_i, e\rangle \beta_i(t), \quad e \in K,$$

and $B_t = B_t^{\omega}$, where B_t^{ω} is the sigma algebra generated by $\{\omega(s): 0 \leq s \leq t\}$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(K_0, H)$ denote the space of all Hilbert–Schmidt operators from K_0 into H. It turns out to be a separable Hilbert space, equipped with the norm

$$\|\psi\|^2_{\mathcal{L}_2^0} = \mathrm{tr}((\psi Q^{1/2})(\psi Q^{1/2})^*) \quad \text{for all } \psi \in \mathcal{L}_2^0.$$

Clearly, for any bounded operators $\psi \in \mathcal{L}(K, H)$, this norm reduces to $\|\psi\|^2_{\mathcal{L}_2^0} = \mathrm{tr}(\psi Q \psi^*)$.

Let $R^+ = [0, +\infty)$ and $C(X, Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, $C \triangleq C([-\tau, 0], R)$ denotes the family of all continuous R-valued functions ϕ defined on $[-\tau, 0]$ with the norm $\|\phi\|_\tau = \sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$, where τ is a positive constant. Denote by $C_H = C([-\tau, 0], H)$ equipped with the norm $\|\phi\|_{C_H} = \sup_{-\tau\leq s\leq 0}\|\phi\|_H$. $\mathrm{sgn}(\cdot)$ is the sign function defined on R.

$PC(J, R^n) = \{\varphi: J \to R^n$ is continuous for all but at most a finite number of points $t \in J$ and at these points $t \in J$, $\varphi(t^+)$ and $\varphi(t^-)$ exist, $\varphi(t^+) = \varphi(t)\}$, where $J \subset R$ is a bounded interval, $\varphi(t^+)$ and $\varphi(t^-)$ denote the right-hand and left-hand limits of the function $\varphi(t)$, respectively. Especially, let $PC \triangleq PC([-\tau, 0], H)$.

Let $PC^b_{\mathscr{F}_0}([-\tau, 0], H)(PC^b_{\mathscr{F}_t}([-\tau, 0], H))$ denotes the family of all bounded $\mathscr{F}_0$ $(\mathscr{F}_t)$-measurable, $PC([-\tau, 0], H)$-value random variables ϕ, satisfying $\|\phi\|^p_{Lp} = \sup_{-\tau\leq\theta\leq 0}\mathbb{E}\|\phi(\theta)\|^p_H < \infty$ for $p > 0$.