Statistics and Probability Letters 79 (2009) 1581–1584

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Weak type inequalities on Lorentz martingale spaces

Yong Jiao [a,b], Lihua Peng [a,*], Peide Liu [b]

[a] School of Mathematical Science and Computing Technology, Central South University, Changsha, 410081, PR China
[b] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China

ARTICLE INFO

Article history:
Received 27 July 2008
Received in revised form 19 March 2009
Accepted 20 March 2009
Available online 31 March 2009

MSC:
primary 60G42
secondary 60G46

ABSTRACT

By the atomic decomposition of Lorentz martingale spaces $H^s_{p,q}$, we obtain the following weak type inequalities for maximal operator and square operator

$$P(Mf>\lambda)\preceq\left(\frac{\|f\|_{H^s_{p,q}}}{\lambda}\right)^p,\ P(Sf>\lambda)\preceq\left(\frac{\|f\|_{H^s_{p,q}}}{\lambda}\right)^p,\quad 0<p<2,\ p<q<\infty.$$

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1. Notations and preliminaries

Let (Ω,Σ,P) be a complete probability space and f a measurable function defined on Ω. The decreasing rearrangement of f is the function f^* defined by

$$f^*(t)=\inf\{s>0:P(|f|>s)\le t\},\quad \forall t>0.$$

We adopt the convention $\inf\emptyset=\infty$. The Lorentz space $L_{p,q}(\Omega)=L_{p,q}$, $0<p<\infty$, $0<q\le\infty$, consists of those measurable functions f with finite quasinorm $\|f\|_{p,q}$ given by

$$\|f\|_{p,q}=\left(\frac{q}{p}\int_0^\infty[t^{1/p}f^*(t)]^q\frac{\mathrm{d}t}{t}\right)^{1/q},\quad 0<q<\infty,$$
$$\|f\|_{p,\infty}=\sup_{t>0}t^{1/p}f^*(t),\quad q=\infty.$$

It will be convenient for us to use an equivalent definition of $\|f\|_{p,q}$, namely

$$\|f\|_{p,q}=\left(q\int_0^\infty[tP(|f(x)|>t)^{1/p}]^q\frac{\mathrm{d}t}{t}\right)^{1/q},\quad 0<q<\infty,$$
$$\|f\|_{p,\infty}=\sup_{t>0}tP(|f(x)|>t)^{1/p},\quad q=\infty.$$

To check that these two expressions are the same, simply make the substitution $s=P(|f(x)|>t)$ and then integrate by parts. We also adopt the notation $L_{p,\infty}=wL_p$. For all these properties on Lorentz spaces, see for example Hunt (1966) or Bennett and Sharply (1988).

* Corresponding author.
E-mail address: lihuapeng125117@sina.com (L. Peng).

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doi:10.1016/j.spl.2009.03.025

Let $\{\Sigma_n\}_{n\geq 0}$ be a non-decreasing sequence of sub-σ-fields of Σ such that $\Sigma = \bigvee \Sigma_n$. We denote the expectation operator and the conditional expectation operator relative to Σ_n by E and E_n, respectively. For a martingale $f = (f_n)_{n\geq 0}$, we define $\Delta_n f = f_n - f_{n-1}$, $n \geq 0$ (with convention $f_{-1} = 0$, $\Sigma_{-1} = \{\Omega, \Phi\}$) and adopt the notions of conditional square function:

$$s_n(f) = \left(\sum_{i=0}^{n} E_{i-1}|\Delta_i f|^2\right)^{1/2}, \qquad s(f) = \left(\sum_{n=0}^{\infty} E_{n-1}|\Delta_n f|^2\right)^{1/2}.$$

As usual, we define Lorentz martingale spaces $H^s_{p,q}$ (see Weisz (1994)),

$$H^s_{p,q} = \{f = (f_n)_{n\geq 0} : \|f\|_{H^s_{p,q}} = \|s(f)\|_{p,q} < \infty\}.$$

Remark. If put $p = q$, we get usual martingale Hardy space H^s_p (see Long (1993)).

The idea of atomic decomposition in martingale theory is derived from harmonic analysis. Just as it does in harmonic analysis, the method is a key ingredient in dealing with many problems including martingale inequalities, duality, interpolation and so on. Weisz (1994) gave some atomic decomposition on martingale Hardy spaces and proved many important theorems by atomic decomposition; Weisz (1998) made a further study of atomic decompositions for weak Hardy spaces consisting of Vilenkin martingale, and proved a weak version of the Hardy–Littlewood inequality by using atomic decomposition; Hou and Ren (2007) considered the vector-valued weak atomic decompositions and weak martingale inequalities; Jiao et al. (2007) discussed the operator interpolation by atomic decompositions of weighted martingale Hardy spaces. In this paper we present an atomic decomposition theorem for Lorentz martingale spaces $H^s_{p,q}$. By the atomic decomposition, we investigate the weak type inequalities for a sublinear operator defined on Lorentz martingale space $H^s_{p,q}$. We start by the definition of atom.

Definition (*Weisz, 1994*). A measurable function a is called a $(1, p, \infty)$-atom if there exists a stopping time τ such that

(i) $a_n = E_n a = 0, \forall n \leq \tau$,

(ii) $\|s(a)\|_\infty \leq P(\tau < \infty)^{-\frac{1}{p}}$.

Throughout the paper, we denote the set of integers and the set of non-negative integers by Z and N, respectively. We write $A \preceq B$ if $A \leq cB$ for some positive constant c independent of appropriate quantities involved in the expressions A and B.

2. Atomic decomposition

Now we can present the atomic decomposition.

Theorem 1. *If the martingale $f \in H^s_{p,q}$, $0 < p < \infty$, $0 < q < \infty$ then there exist a sequence (a^k) of $(1, p, \infty)$-atoms and a real number sequence $(\mu_k) \in l_q$ such that*

$$f_n = \sum_{k\in Z} \mu_k a^k_n, \quad \forall n \in N$$

and

$$\|\mu_k\|_{l_q} \preceq \|f\|_{H^s_{p,q}}.$$

Proof. Assume that $f \in H^s_{p,q}$. Now considering the following stopping time for all $k \in Z$:

$$\tau_k = \inf\{n \in N : s_{n+1}(f) > 2^k\} (\inf \phi = \infty).$$

The sequence of these stopping times is obviously non-decreasing. It easy to see that

$$\sum_{k\in Z}(f_n^{\tau_{k+1}} - f_n^{\tau_k}) = \sum_{k\in Z}\left(\sum_{m=0}^{n} \chi_{\{m\leq\tau_{k+1}\}}\Delta_m f - \sum_{m=0}^{n}\chi_{\{m\leq\tau_k\}}\Delta_m f\right)$$
$$= \sum_{k\in Z}\left(\sum_{m=0}^{n}\chi_{\{\tau_k<m\leq\tau_{k+1}\}}\Delta_m f\right) = f_n.$$

Let

$$\mu_k = 2^k 3P(\tau_k < \infty)^{\frac{1}{p}},$$

and

$$a^k_n = \frac{f_n^{\tau_{k+1}} - f_n^{\tau_k}}{\mu_k}.$$

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