Available online at www.sciencedirect.com
ScienceDirect
Statistics & Probability Letters 77 (2007) 964–972
STATISTICS & PROBABILITY LETTERS
www.elsevier.com/locate/stapro

Fisher information in record values and their concomitants about dependence and correlation parameters

Morteza Amini, J. Ahmadi*,[1]

Department of Statistics, School of Mathematical Sciences, Ferdowsi University of Mashhad, P.O. Box 91775-1159, Mashhad, Iran

Received 15 December 2005; received in revised form 4 December 2006; accepted 17 January 2007
Available online 9 February 2007

Abstract

Let $\{(X_i, Y_i), i \geqslant 1\}$ be a sequence of bivariate random variables from a continuous distribution with single real valued parameter θ. In this paper, we investigate the properties of Fisher information about the dependence and correlation parameters in the sequence of the first n records and their concomitants and compare it with the desired information in an i.i.d. sample of size n from a bivariate distribution. Under the assumption that the marginal distribution of X is free of θ the additivity property of the Fisher information is investigated. An explicit expression of Fisher information in record values and their concomitants is given for the Farlie–Gumbel–Morgenstern (FGM) copula family which are parameterized by dependence parameter. It is shown that the Fisher information contained in record values and their comcomitants is more than that of the same number of i.i.d. bivariate observations from FGM family of distributions. The relative efficiency (RE) of that estimator of θ whose variance is equal to Cramér–Rao lower bound, based on record values and their concomitants and i.i.d. observations are studied. Similar results are obtained for bivariate normal in the case that θ is correlation parameter. Finally some numerical results for the corresponding RE for the estimators of Kendall's correlation parameter, tau, are given for one of the most common families of Archimedean Copulas, namely Gumbel–Hougaard model.

MSC: 62G30; 62B10; 62F10; 62F12

Keywords: Archimedean copulas; Bivariate normal distribution; Farlie–Gumbel–Morgenstern family; Gumbel–Hougaard model; Relative efficiency

1. Introduction

Recently, the problem of finding the efficient estimators based on record values, caused the researchers to take the evaluation of the corresponding Fisher information into concern. In order to find an answer for the question "How much information contained in record values?", Ahmadi and Arghami (2001), Stepanov et al. (2003) and Balakrishnan and Stepanov (2005) have been working on the Fisher information contained in

*Corresponding author. Fax: +98 51 18828605.
E-mail addresses: mort_amini@gmail.com (M. Amini), ahmadi@math.um.ac.ir (J. Ahmadi).
[1]The second author is a member of Statistics Center of Excellence of Ferdowsi University of Mashhad.

0167-7152/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.spl.2007.01.003

a sequence of record values (and weak records) and comparing it with the desired information in an i.i.d. sample. Ahmadi and Arghami (2003), Hofmann and Nagaraja (2003), Hofmann and Balakrishnan (2004) and Hofmann (2004) also considered the additional information in record times given the record values. The Fisher information plays an important role in statistical estimation and inference through the information (Cramér–Rao) inequality and its association with the asymptotic properties, specially the asymptotic variance of the maximum likelihood estimator (MLE). It also can be used to compute the relative efficiency (RE) of the estimator with a variance equal to Cramér–Rao lower bound, which we call it the optimal estimator (OE), i.e., $Var(\delta(\mathbf{X})) = ((\partial/\partial\theta)E\delta(\mathbf{X}))^2/I_{\mathbf{X}}(\theta)$.

Now let $\{(X_i, Y_i), i \geqslant 1\}$ be a sequence of bivariate random variables from a continuous distribution with parameter θ. If we place the X-values in ascending order, then the corresponding Y-value of each order statistic is termed the concomitant of that order statistic. There are several estimation problems based on concomitants of order statistics in the literature that use them as a tool for deriving "quick" estimations of correlation coefficient (e.g. Watterson, 1959; Barnett et al., 1976 and recently Modarres and Zheng, 2004), regression coefficient (e.g. Barton and Casley, 1958) and other parameters (e.g. Harrell and Sen, 1979; Gill et al., 1990). Recently Abo-Eleneen and Nagaraja (2002) investigate some properties of Fisher information in an order statistic and its concomitant.

There is a similar concept of concomitants for the record statistics. Let us denote the sequence of record values of the first component by $\{R_n\}_{n\geqslant 0}$. The second component associated with R_n is termed the concomitant of nth record value and denoted by $R_{[n]}$. The reader is referred to Raqab and Ahsanullah (2002) for a general review of concomitants of ordered random variables. The most important use of concomitants of record values arises in experiments in which a specified characteristic's measurements of an individual are made sequentially and only values that exceed or fall below the current extreme value are recorded. So the only observations are record values and their associated other characteristics which are called the concomitants of that record values. Such situations often occur in industrial stress, life time experiments, sporting matches, weather data recording and some other experimental fields. The other important application is in life testing problems in which testing of an item is destructive and costly. If the items are expensive, one can set up the experiment so that only the "low life" units are destroyed. As an example, one may consider the example of testing the breaking strength of wooden beams cited in Gulati and Padgett (1995) and Glick (1978), in which beams are replaced when they do not break until the pressure reaches the minimum previously observed breaking time. In other words, only the lower record values are observed. Now suppose that we want to fit a model for the relationship between the breaking strength as the response variable and some other characteristics of wooden beams as predictors. Here our only observations are the lower records of breaking strength and their associated concomitants. In such situations the problem of "estimation based on record values and their concomitants" is in a high degree of importance. This problem leads us to the well-known question "How much information is contained in record values and their concomitants about a specified parameter of the population?". This question motivates one to investigate the Fisher information contained in record values and their concomitants. In this paper, we investigate the properties of Fisher information in the sequence of the first n records and their concomitants. As the definition of concomitants is based on the dependence, one may be interested to investigate about the Fisher information properties of the sequence of record values and their concomitants about the dependence and correlation parameters. Copula models play an essential role in dependence modeling of multivariate and specially bivariate distributions. This approach is rooted in a representation theorem due to Sklar (1959). The main advantage of this approach is that the selection of an appropriate model for the dependence between X and Y, represented by the Copula, can be proceeded independently from the choice of marginal distributions. Therefore, we can investigate the Fisher information properties of the dependence parameters independently from other parameters.

Section 2 contains some preliminaries and also under the assumption that the marginal distribution of X is free of θ, the additivity property of the Fisher information is investigated. In Section 3, explicit expression of Fisher information in record values and their concomitants is given for the FGM family of distributions which are parameterized by dependence parameter. It is shown that Fisher information contained in record values and their concomitants is more than that of the same number of i.i.d. bivariate observations from FGM family of distributions. The RE of the OE of θ based on record values and their concomitants and i.i.d. observations