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Ruin problems in risk models with dependent rates of interest ☆

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Abstract

In this paper, we consider ruin problems in two generalized risk models. The effects of timing of payments and interest on the ruin problems in the models are studied. The rates of interest $\{I_n, n = 1, 2, \cdots\}$ are assumed to have an autoregressive structure. We obtain the recursive formulas of penalty functions which give a unified treatment to ruin quantities including the distribution of surplus immediately before ruin and the deficit at ruin, etc. Furthermore, we consider the probability properties of the duration of ruin, which are used to describe the severity of ruin.

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Keywords: Rate of interest; Penalty function; Duration of ruin

1. Introduction

Consider the following two risk models, in which the effects of timing of payments and interest on the surplus process and on the ruin probability can be included.

$$U_n = (U_{n-1} + X_n)(1 + I_n) - Y_n \tag{1}$$

and

$$U_n = U_{n-1}(1 + I_n) + X_n - Y_n, \tag{2}$$

where the claim sizes $\{Y_n, n = 1, 2, \cdots\}$ and the premiums $\{X_n, n = 1, 2, \cdots\}$ are two i.i.d. nonnegative sequences of random variables and the rates of interest $\{I_n, n = 1, 2, \cdots\}$ are assumed to have a dependent autoregressive structure of order 1, i.e. I_n satisfies

$$I_n = \alpha I_{n-1} + W_n, \quad n = 1, 2, \cdots, \tag{3}$$

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where, $0 \leqslant \alpha < 1$ and $I_0 = i_0 \geqslant 0$ are constant and $\{W_n, n = 1, 2, \cdots\}$ is a sequence of i.i.d. nonnegative random variables. Futhermore, $\{Y_n, n = 1, 2, \cdots\}$, $\{X_n, n = 1, 2, \cdots\}$ and $\{W_n, n = 1, 2, \cdots\}$ are assumed to be independent, and they have common distribution functions $G(y) = \Pr\{Y_1 < y\}$, $F(x) = \Pr\{X_1 < x\}$, $H(w) = \Pr\{I_1 < w\}$, respectively.

Cai (2002) investigated the ruin probabilities of models (1) and (2). Apart from the ruin probability, other important ruin quantities in ruin theory include the Laplace transform of the time of ruin; the surplus immediately before ruin; etc. A unified method to study these ruin quantities is to consider the (expected discounted) penalty function associated with the time of ruin. To simplify notation, we use the common notation T to denote the time of ruin at which U_n is less than 0 for the first time in models (1) and (2), i.e. $T \triangleq T(u, i_0) = \inf\{n : U_n < 0\}$. Let $\Phi_\beta^{(j)}(u, i_0), j = 1, 2$ denote the penalty function of models (1) and (2), respectively, i.e.

$$\Phi_\beta^{(j)}(u, i_0) = E[g(U_{T-}, |U_T|)e^{-\beta T} I(T < \infty) | U_0 = u, I_0 = i_0], \quad j = 1, 2, \tag{4}$$

where $g(x, y), x \geqslant 0, y \geqslant 0$, is a nonnegative function such that $\Phi_\beta^{(j)}(u, i_0)$ $(j = 1, 2)$ exists; $\beta \geqslant 0$ and $I(C)$ is the indicator function of a set C. Related discussions can be found in Yang (1998a,b), Gerber and Shiu (1997), Cai and Dickson (2002) and Cai (2004).

Furthermore, the duration of ruin is also an important ruin quantity, which can be used to reflect the severity of ruin. Here, we define

$$\tau(u, i_0) = \inf\{n : n > T(u, i_0), U_n > 0\}, \tag{5}$$

where $\tau(u, i_0)$ denotes the first time at which the risk process U_n crosses above 0 after T. Hence, the duration of ruin is defined:

$$\widetilde{T}(u, i_0) = \begin{cases} \tau(u, i_0) - T(u, i_0), & T(u, i_0) < \infty, \\ 0, & T(u, i_0) = \infty. \end{cases} \tag{6}$$

Write

$$\psi_k^{(j)}(u, i_0) \triangleq P\{\widetilde{T}(u, i_0) = k\}, \quad j = 1, 2, \tag{7}$$

where $\psi_k^{(j)}(u, i_0), j = 1, 2$ denotes the probability that the duration of ruin is k, respectively, in models (1) and (2).

In this paper, we give the recursive formulas of penalty functions and consider the probability properties of the duration of ruin when the interest process $\{I_n, n = 1, 2, \cdots\}$ has a dependent autoregressive structure of order 1 in models (1) and (2).

2. Integral equations for penalty functions

In this section, we will give recursive formulae of the penalty functions in (1) and (2).

It is easily seen that (1) is equivalent to

$$U_n = u \prod_{k=1}^{n} (1 + I_k) + \sum_{k=1}^{n} \left((X_k(1 + I_k) - Y_k) \prod_{t=k+1}^{n} (1 + I_t) \right), \tag{8}$$

while (2) implies that

$$U_n = u \prod_{k=1}^{n} (1 + I_k) + \sum_{k=1}^{n} \left((X_k - Y_k) \prod_{t=k+1}^{n} (1 + I_t) \right), \tag{9}$$

where $\prod_a^b (1 + I_t) = 1$, for $a > b$.