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Minimum distance index for complex valued ICA

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ABSTRACT

We generalize the Minimum Distance (MD) index to be applicable in complex valued ICA. To illustrate the use of the MD index, we present a complex version of AMUSE and compare it to complex FOBI in a simulation study.

1. Introduction

In the complex valued independent component (IC) model, the elements of a p-variate random vector are assumed to be linear combinations of the elements of an unobservable complex valued p-variate vector with mutually independent components. In complex valued independent component analysis (ICA) the aim is to recover the independent components by estimating a complex valued unmixing matrix that transforms the observed p-variate vector to mutually independent components. The IC model is not in general uniquely defined as after permutation, scaling and phase shifts, independent components remain independent. Thus, different ICA estimates can estimate different population quantities. ICA is applied in various fields of science, e.g. biomedical image data applications, signal processing and economics, see Hyvärinen et al. (2001) and Comon and Jutten (2010). Complex random signals play an increasingly important role in the field of ICA. The complex IC model is used for example in magnetic resonance imaging and antenna array signal processing for wireless communications and radar applications. See for example Hyvärinen et al. (2001) and Ollila et al. (2008).

The main contribution of this paper is that we extend the minimum distance (MD) index for complex valued ICA. The MD index was presented in the case of real valued ICA in Ilmonen et al. (2010). The index is independent of the standardization of the model and thus treats different ICA estimators fairly even if they estimate different quantities in terms of order, scale

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and phase. We present a generalization of the MD index for complex valued ICA. We also consider some properties of the complex MD index. Additionally, we present a complex valued version of the well-known AMUSE functional. The AMUSE transformation, for real valued time series ICA, is based on simultaneous use of the real valued covariance matrix and a real valued autocovariance matrix with lag $\tau \neq 0$. The method was presented in Tong et al. (1990), and its asymptotic behavior was examined in Miettinen et al. (2012). In this article, we consider the AMUSE functional in the case of complex valued time series. We conduct a simulation study to illustrate the use of the generalized MD index by comparing the complex AMUSE and complex FOBI (Cardoso, 1989; Ilmonen, 2013) estimators in different settings.

The paper is organized as follows. In Section 2 we generalize the MD index for complex valued ICA. In Section 3 we present the complex version of the AMUSE functional. Simulations are presented in Section 4 and some final remarks are given in Section 5. The proofs of the theorems are presented in the Appendix.

2. Minimum distance index for complex valued ICA

The basic complex valued independent component (IC) model has the form,

$$x_t = \Omega z_t + \mu, \quad t = 1, \ldots, n, \tag{1}$$

where Ω is a full-rank $p \times p$ complex valued mixing matrix, z_t is an unobservable centered complex-valued p-vector with mutually independent components and μ is a p-variate location vector. The location μ is usually a nuisance parameter that is in the following assumed to be zero for simplicity. Based on the observed p-vector x_t the goal is to find an unmixing matrix Γ such that Γx_t has again mutually independent components. Clearly the model is ill-defined as order, scale and phase of the components of z_t are not well-defined.

The literature, especially in the real case, suggests many ICA estimators, $\hat{\Gamma}$, and their performance is usually compared using performance indices. For a review in the real case about different indices see Nordhausen et al. (2011). Most of the indices presented in the literature assume that the IC model is standardized in some particular way, see Ilmonen et al. (2010). For example, the popular AMARI index (Amari et al., 1996) that can be used for complex ICA as well, see Ollila et al. (2008), suffers from this problem as it assumes that the methods assign the components the same scale, Ilmonen et al. (2010). This can be problematic as different estimators may estimate different scales. The MD index, introduced in Ilmonen et al. (2010) for real valued ICA, was designed to solve this problem. We next generalize the MD index for complex valued ICA.

Let $\mathcal{C}$ denote the set of matrices of the form DP, where P is a $p \times p$ permutation matrix and D is a complex valued $p \times p$ diagonal matrix. The sets $\{CA : C \in \mathcal{C}\}$ partition the set of complex valued $p \times p$ matrices into equivalence classes. If $B \in \{CA : C \in \mathcal{C}\}$, notation $A \sim B$ is used. The shortest squared distance between the set $\{CA : C \in \mathcal{C}\}$ of matrices that are equivalent to A and I_p is given by

$$D^2(A) = \frac{1}{p-1} \inf_{C \in \mathcal{C}} \left\| CA - I_p \right\|_F^2, \tag{2}$$

where $\|\cdot\|_F$ is the Frobenius norm.

Remark 2.1. Note that $D^2(A) = D^2(CA)$ for all $C \in \mathcal{C}$.

Theorem 2.1. *Let A be any complex valued $p \times p$ full rank matrix. The shortest squared distance $D^2(A)$ fulfills the following four conditions given below:*

1. $1 \geq D^2(A) \geq 0$,
2. $D^2(A) = 0$ *iff* $A \sim I_p$,
3. $D^2(A) = 1$ *iff* $A \sim 1_p a^T$ *for some complex valued p-variate vector a, and*
4. *the function* $c \to D^2\left(I_p + c \text{ off } (A)\right)$ *is increasing in* $c \in [0, 1]$ *for all matrices A such that* $\left|A_{ij}\right| \leq 1, i \neq j$.

Consider the complex valued IC model with mixing matrix Ω and an unmixing matrix estimate $\hat{\Gamma}$. The shortest distance between the identity matrix and the set of matrices $\left\{C\hat{G} : C \in \mathcal{C}\right\}$ equivalent to the gain matrix $\hat{G} = \hat{\Gamma}\Omega$ is given in the following definition.

Definition 2.1 (*Minimum Distance Index*)**.** The minimum distance index for $\hat{\Gamma}$ is

$$\hat{D} = D(\hat{\Gamma}\Omega) = \frac{1}{\sqrt{p-1}} \inf_{C \in \mathcal{C}} \left\| C\hat{\Gamma}\Omega - I_p \right\|_F.$$

It follows from Theorem 2.1 that $1 \geq \hat{D} \geq 0$, and $\hat{D} = 0$ only if $\hat{\Gamma} \sim \Omega^{-1}$. Furthermore, $\hat{D} = 1$ is obtained in the pathological case when all the row vectors of $\hat{\Gamma}\Omega$ have the same direction. The value of the minimum distance index is now easy to interpret. Values close to 0 are associated with excellent separation, and large values indicate poor performance.