



Optimal reinsurance with both proportional and fixed costs

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ABSTRACT

This paper investigates an optimal reinsurance problem with both proportional and fixed transaction costs. Associated with a reinsurance, the proportional cost is usually charged by the reinsurer and the fixed cost occurs at the beginning of the reinsurance due to consultant commission. We describe an insurer's original cash reserve process by a Brownian motion with positive drift. The insurer aims to minimize its ruin probability by taking a dynamic reinsurance strategy. This leads to a mixed regular control and optimal stopping problem. We solve it by establishing a connection with an optimal stopping problem. The value function and the optimal reinsurance strategy are obtained explicitly.

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1. Introduction

In practice, reinsurance is an important way for an insurer to control its risk exposure. Therefore, in the past few decades, the optimal reinsurance problem has been extensively studied for various objective functions in mathematical finance literature. The popular objective functions include, but not limit to, minimizing ruin probability and maximizing present value of dividend payments. To name a few, Browne (1995), Schmidli (2001), Zhang et al. (2007) and Promislow and Young (2005) studied the optimal (proportional, excess-of-loss, or other) reinsurance by minimizing the ruin probability; Taksar (2000) and Højgaard and Taksar (2004) considered the optimal reinsurance problem by maximizing the firm value, that is the expected present value of the dividends paid out up to the time of ruin; while Højgaard and Taksar (1998) studied optimal proportional reinsurance with transaction costs by maximizing the expected present value of cash reserve process up to the ruin time. In addition, other controls like investment and capital injections are also involved in the optimal reinsurance problem, for example, see Zhou and Yuen (2012), Shen and Zeng (2014), Bai and Guo (2008) and Zeng and Li (2011).

In previous studies, it is usually assumed that reinsurance is non-cheap, that is the reinsurer requires more premium (with a higher loading) than the insurer for a given risk. So, the extra premium can be seemed as the proportional costs associated with the reinsurance arrangement. In fact, except for the proportional costs, taking reinsurance also occurs a fixed cost for the insurer at the start time because of consultant commission. However, the fixed cost is ignored by nearly all the studies except for Egami and Young (2009).

In this paper, we investigate a dynamic optimal reinsurance problem in presence of both proportional and fixed costs. Assume that an insurer aims to minimize the ruin probability by taking a non-cheap proportional reinsurance. Also assume that the reinsurance is irreversible, and a fixed cost occurring at the beginning of reinsurance. With these assumptions,

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the optimal reinsurance problem is formulated to be a mixed regular control and optimal stopping problem for a diffusion model. We solve it by establishing a connection with an optimal stopping problem. More specifically, we first consider two alternative strategies: (i) never taking reinsurance and (ii) taking reinsurance immediately. The optimal reinsurance strategy is shown to be never taking reinsurance if the proportional costs is large, and waiting to take the reinsurance when the surplus hits a level. The explicit expression for the survival probability under the optimal reinsurance strategy is obtained and it is larger than that with the above strategies.

The remainder of this paper is as follows. Section 2 describes the optimal reinsurance problem, gives the dynamics of the cash process and analyzes some useful benchmarks. Section 3 provides a formulation of the optimization problem based on the dynamic programming principle and solves the optimal reinsurance problem in terms of an optimal stopping problem. The optimal reinsurance strategy and the value function are also obtained in Section 3. Finally, we present an example to give some explanation on the results in Section 4.

2. The model

We start with a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$, where the filtration $\{\mathcal{F}_t\}$ satisfies the usual conditions. $W = \{W_t; t \geq 0\}$ is a standard Brownian motion with respect to $\{\mathcal{F}_t\}$. Let $\mathcal{G}$ denote the set of positive non-decreasing right-continuous processes and $\mathcal{T}$ the set of $\mathcal{F}_t$ -stopping times. Throughout the paper, it is assumed that all stochastic processes and random variables are well-defined on this probability space.

2.1. Formulation of the problem

Now consider a large insurance company whose cash flow is described by a diffusion process, i.e.,

$$dR_{1,t} = \mu_1 dt + \sigma_1 dW_t, \quad R_{1,0} = x, \quad (2.1)$$

where $x \geq 0$ is the initial cash reserve, $\mu_1 > 0$ is the profit rate and $\sigma_1 > 0$ is the volatility. Assume that the insurer takes a proportional reinsurance to control its risk exposure and the reinsurance is non-cheap. Then, with a reinsurance strategy, the cash flow process can be expressed as

$$dR_{2,t}^a = \mu_2(a_t)dt + \sigma_2(a_t)dW_t, \quad R_{2,0}^a = x, \quad (2.2)$$

with

$$\mu_2(a_t) = a_t \mu_1 - \kappa(1 - a_t)\mu_1, \quad (2.3)$$

$$\sigma_2(a_t) = a_t \sigma_1, \quad (2.4)$$

where $0 \leq a_t \leq 1$ is the proportion of total risk retained by the insurer, and κ is the proportional cost rate for the reinsurance. When $\kappa = 0$, the reinsurance is called cheap reinsurance, and it is non-cheap when $\kappa > 0$. To avoid of a trivial case, we assume $\kappa > 0$ in this paper. In addition, we also assume that there exists a fixed cost, say $I \geq 0$, at the beginning of the reinsurance.

Within this framework, the insurer aims to minimize the retained risk by choosing the start time of the reinsurance and dynamically adjusting risk exposure a_t . Here we take ruin probability as a risk measure for the insurer. Then, the optimal reinsurance problem can be formulated as follows.

We use $\pi = (\tau^\pi, a_t^\pi; t \geq 0)$ to denote a reinsurance strategy and it is said to be admissible if

- τ^π belongs to $\mathcal{T}$;
- $\{a_t^\pi\}_{t \geq 0}$ is a $\mathcal{F}_t$ -predictable stochastic process such that $0 \leq a_t \leq 1$ for all $t \geq 0$.

We denote the set of all admissible controls by Π . The control component τ^π corresponds to the start time of reinsurance and the control component a_t^π represents the proportion of risk exposure at time t . For a given $\pi = (\tau^\pi, a_t^\pi; t \geq 0) \in \Pi$, the associated cost process $\{I_t^\pi\}_{t \geq 0}$ can be fully characterized by

$$I_t = I \mathbf{1}_{\{t \geq \tau^\pi\}},$$

where $I \geq 0$ is a constant and $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function. With the strategy π , the dynamics of the cash process, denoted by $\{R_t^\pi\}$, satisfies

$$dR_t^\pi = [\mu_1 \mathbf{1}_{\{t < \tau^\pi\}} + \mu_2(a_t^\pi) \mathbf{1}_{\{t \geq \tau^\pi\}}] dt + [\sigma_1 \mathbf{1}_{\{t < \tau^\pi\}} + \sigma_2(a_t^\pi) \mathbf{1}_{\{t \geq \tau^\pi\}}] dW_t - dI_t^\pi, \quad R_0^\pi = x \quad (2.5)$$

where $\mu_2(\cdot)$ and $\sigma_2(\cdot)$ are given by (2.3) and (2.4) respectively. Correspondingly, we define the time of ruin by

$$T^\pi = \inf\{t \geq 0 : R_t^\pi \leq 0\}.$$

Then, the survival probability is defined by

$$\delta^\pi(x) = \mathbb{P}(T^\pi = \infty | R_0^\pi = x). \quad (2.6)$$

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