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On large deviations of extremes under power normalization[☆]

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ABSTRACT

By using the theory of p-max stable laws, we study large deviations of the extremes under power normalization. The necessary and sufficient conditions for large deviations of each of the six p-max stable laws are derived.

1. Introduction

Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables with common distribution function (d.f.) F. Define $M_n = \max(X_1, \ldots, X_n)$. A d.f. F is said to belong to the max domain of attraction of a nondegenerate d.f. G under linear normalization (notation: $F \in D_l(G)$) if there exist constants $a_n > 0$ and $b_n \in \mathbb{R}$, $n \geq 1$, such that

$$\lim_{n\to\infty} P\Big(\frac{M_n - b_n}{a_n} \leq x\Big) = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x)$$

at all points $x \in C(G)$, the set of continuity points of G. It is well known that G can be only one of the three types of extreme value d.f.'s, which were called as l-max stable laws in Mohan and Ravi (1993), namely:

$$\Phi_\alpha(x) = \begin{cases} 0, & \text{if } x < 0, \\ \exp\{-x^{-\alpha}\}, & \text{if } x \geq 0, \end{cases}$$

$$\Psi_\alpha(x) = \begin{cases} \exp\{-(-x)^{\alpha}\}, & \text{if } x < 0, \\ 1, & \text{if } x \geq 0, \end{cases}$$

$$\Lambda(x) = \exp\{-e^{-x}\}, \quad x \in \mathbb{R},$$

where α is a positive parameter. These three types can be summarized by the following representation:

$$G_\gamma(x) := \exp(-(1+\gamma x)^{-1/\gamma}), \quad 1 + \gamma x \geq 0,$$

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for $\gamma \in \mathbb{R}$. Criteria for $F \in D_l(G)$ and the choice of normalizing constants, a_n and b_n, can be found in de Haan and Ferreira (2006), Leadbetter et al. (1983) and Resnick (1987).

A d.f. F is said to belong to the max domain of attraction of a nondegenerate d.f. H under power normalization (notation: $F \in D_p(H)$) if there exist constants $\alpha_n > 0$ and $\beta_n > 0$, $n \geq 1$, such that

$$\lim_{n\to\infty} P\left(\left|\frac{M_n}{\alpha_n}\right|^{1/\beta_n} \operatorname{sign}(M_n) \leq x\right) = \lim_{n\to\infty} F^n(\alpha_n \mid x \mid^{\beta_n} \operatorname{sign}(x)) = H(x)$$

at all points $x \in C(H)$, where $\operatorname{sign}(x) = -1, 0$ or 1 according as $x < 0, = 0$ or > 0, respectively. It is known (see (Pancheva, 1985; Mohan and Ravi, 1993) that H can be only one of the six p-types of d.f.'s, which were called as p-max stable laws in Mohan and Ravi (1993), namely: for some $\alpha > 0$,

$$H_{1,\alpha}(x) = \begin{cases} 0, & \text{if } x \leq 1, \\ \exp\{-(\log x)^{-\alpha}\}, & \text{if } x > 1, \end{cases}$$

$$H_{2,\alpha}(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ \exp\{-(-\log x)^{\alpha}\}, & \text{if } 0 < x < 1, \\ 1, & \text{if } x \geq 1, \end{cases}$$

$$H_{3,\alpha}(x) = \begin{cases} 0, & \text{if } x \leq -1, \\ \exp\{-(-\log(-x))^{-\alpha}\}, & \text{if } -1 < x < 0, \\ 1, & \text{if } x \geq 0, \end{cases}$$

$$H_{4,\alpha}(x) = \begin{cases} \exp\{-(-\log(-x))^{\alpha}\}, & \text{if } x < -1, \\ 1, & \text{if } x \geq -1, \end{cases}$$

$$\Phi(x) = \Phi_1(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ \exp\{-x^{-1}\}, & \text{if } x > 0, \end{cases}$$

$$\Psi(x) = \Psi_1(x) = \begin{cases} \exp\{x\}, & \text{if } x < 0, \\ 1, & \text{if } x \geq 0. \end{cases}$$

Two distributions $H_1(x)$ and $H_2(x)$ are of the same p-type if there exist some constants $A > 0$ and $B > 0$ such that

$$H_1(x) = H_2(A|x|^B \operatorname{sign}(x))$$

for all $x \in \mathbb{R}$. As in the case of linear normalization, Nasri-Roudsari (1999) summarized the six p-types of the extreme value distributions under power normalization by the following von-Mises type representations:

$$H_\xi^{(1)}(x) = \exp\left(-(1+\xi\log x)^{-1/\xi}\right), \quad \xi \in \mathbb{R},\ x > 0,\ 1+\xi\log x \geq 0,$$

for the three p-types $H_{1,\alpha}$, $H_{2,\alpha}$ and Φ if $\xi < 0$, $\xi > 0$ and $\xi = 0$, respectively, and

$$H_\xi^{(2)}(x) = \exp\left(-(1-\xi\log(-x))^{-1/\xi}\right), \quad \xi \in \mathbb{R},\ x < 0,\ 1-\xi\log(-x) \geq 0,$$

for the three p-types $H_{3,\alpha}$, $H_{4,\alpha}$ and Ψ if $\xi < 0$, $\xi > 0$ and $\xi = 0$, respectively. Necessary and sufficient conditions for d.f.'s to belong to one of the max domains of attractions of p-max stable laws have been given in Mohan and Subramanya (1991), Mohan and Ravi (1993), Subramanya (1994) and Christoph and Falk (1996). It is known from Mohan and Ravi (1993) that the max stable laws under power normalization attract more distributions than that under linear normalization. Barakat and Nigm (2002) and Peng et al. (2012) established the limiting distributions of extreme order statistics under power normalization with random index. Ravi and Praveena (2010) provided von Mises type sufficient conditions that F belongs to the max domain of attraction of the Frechét or Weibull laws under power normalization. By applying the transformation theorem, Christoph and Falk (1996) studied the relationship between p-max and l-max domains of attraction. Peng et al. (2013) studied moment convergence and density convergence of extremes under power normalization. Large deviations have been studied well under linear normalization, see Resnick (1987). Drees et al. (2003) obtained a general result for large deviations of extremes under second-order condition. As far as we know, there is no work on large deviations of extremes under power normalization. Our objective is to extend large deviations of extremes under linear normalization to power normalization.

The contents of this paper are organized as follows. Some auxiliary results are given in Section 2. Necessary and sufficient conditions for large deviations of extremes under power normalization are obtained in Section 3. Their proofs are deferred to Section 4. In Section 5, we derive large deviations of extremes under the second-order condition.

2. Preliminaries

Let $r(F) = \sup\{x : F(x) < 1\}$ denote the right endpoint of F. For any nondecreasing function F, let $F^{\leftarrow}$ be its left-continuous inverse, i.e., $F^{\leftarrow}(y) = \inf\{x : F(x) \geq y\}$.

In this paper, we discuss large deviations of extremes, i.e., we seek $x_n \uparrow r(F)$ such that

$$\lim_{n\to\infty} P\left(\left|\frac{M_n}{\alpha_n}\right|^{1/\beta_n} \operatorname{sign}(M_n) > y_n\right) \Big/ (1-H(y_n)) = \lim_{n\to\infty} \left(1 - F^n(\alpha_n|y_n|^{\beta_n}\operatorname{sign}(y_n))\right)/(1-H(y_n)) = 1 \tag{2.1}$$

for any sequence $\{y_n\}$ such that $y_n = O(x_n)$. We always assume that $\{x_n\}$ is strictly increasing.