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Precise asymptotics in complete moment convergence of moving-average processes

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Abstract

In this paper, we discuss moving-average process $X_k = \sum_{i=-\infty}^{\infty} a_{i+k}\varepsilon_i$, where $\{\varepsilon_i; -\infty < i < \infty\}$ is a doubly infinite sequence of i.i.d random variables with mean zeros and finite variances, $\{a_i; -\infty < i < \infty\}$ is an absolutely summable sequence of real numbers. Set $S_n = \sum_{k=1}^{n} X_k$, $n \geqslant 1$. Suppose $E|\varepsilon_1|^3 < \infty$, we prove that, if $E|\varepsilon_1|^r < \infty$, for $1 < p < 2$ and $r > 1 + p/2$, then

$$\lim_{\varepsilon \searrow 0} \varepsilon^{2(r-p)/(2-p)-1} \sum_{n=1}^{\infty} n^{r/p-2-1/p} E\{|S_n| - \varepsilon n^{1/p}\}_+ = \frac{p(2-p)}{(r-p)(2r-p-2)} E|Z|^{2(r-p)/(2-p)},$$

where Z has a normal distribution with mean 0 and variance $\tau^2 = \sigma^2(\sum_{i=-\infty}^{\infty} a_i)^2$.

MSC: 60G50; 60F15

Keywords: Moving-average process; Complete moment convergence; Berry–Esseen inequality

1. Introduction and main results

We assume that $\{\varepsilon_i; -\infty < i < \infty\}$ is a doubly infinite sequence of i.i.d random variables with mean zeros and finite variances. Let $\{a_i; -\infty < i < \infty\}$ is an absolutely summable sequence of real numbers and

$$X_k = \sum_{i=-\infty}^{\infty} a_{i+k}\varepsilon_i, \quad k \geqslant 1. \tag{1.1}$$

Under some suitable conditions, many limiting results have been obtained for moving-average process $\{X_k; k \geqslant 1\}$. For example, Burton and Dehling (1990) have obtained a large deviation principle for $\{X_k; k \geqslant 1\}$, Yang (1996) has established the central limit theorem (CLT) and the law of the iterated logarithm, Li et al. (1992a) and Zhang (1996) have obtained the result on complete convergence, etc.

When $\{X_k; k \geqslant 1\}$ is a sequence of i.i.d random variables with common distribution function F, mean 0 and positive, finite variance, Li et al. (1992b) derived convergence rates of moderate deviations and

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the precise asymptotics in the law of the iterated logarithm. Chen (1978) and Gut and Spătaru (2000) studied the precise asymptotics in the Baum–Katz law of large numbers as $\varepsilon \searrow 0$. One of their results is as follows.

Theorem A. *Suppose that* $\{X_k; k \geqslant 1\}$ *is a sequence of i.i.d random variables with* $EX_1 = 0$ *and* $0 < EX_1^2 = \gamma^2 < \infty$. *Then, for* $1 \leqslant p < r$,

$$\lim_{\varepsilon \searrow 0} \varepsilon^{2(r-p)/(2-p)} \sum_{n=1}^{\infty} n^{r/p-2} P\left\{\left|\sum_{k=1}^{n} X_k\right| \geqslant \varepsilon n^{1/p}\right\} = \frac{p}{r-p} E|Z|^{2(r-p)/(2-p)},$$

where Z *has a normal distribution with mean* 0 *and variance* γ^2.

On the other hand, Chow (1988) discussed complete moment convergence of i.i.d random variables. He got

Theorem B. *Suppose that* $\{X_k; k \geqslant 1\}$ *is a sequence of i.i.d random variables with* $EX_1 = 0$. *For* $1 \leqslant p < 2$ *and* $r > p$, *if* $E\{|X_1|^r + |X_1| \log(1 + |X_1|)\} < \infty$, *then for any* $\varepsilon > 0$, *we have*

$$\sum_{n=1}^{\infty} n^{r/p-2-1/p} E\left\{\left|\sum_{k=1}^{n} X_k\right| - \varepsilon n^{1/p}\right\}_+ < \infty.$$

Our purpose of this paper is to show that the precise asymptotics result in this kind of complete moment convergence also holds for moving-average process.

Set $S_n = \sum_{k=1}^{n} X_k,\ n \geqslant 1$ $\{X_k; k \geqslant 1\}$ is defined as (1.1). Our result is as follows.

Theorem 1.1. *Suppose* $\{X_k; k \geqslant 1\}$ *is defined as* (1.1), *where* $\{a_i; -\infty < i < \infty\}$ *is a sequence of real numbers with* $\sum_{i=-\infty}^{\infty} |a_i| < \infty$ *and* $\{\varepsilon_i; -\infty < i < \infty\}$ *is a sequence of i.i.d random variables with* $E\varepsilon_1 = 0, E\varepsilon_1^2 < \infty$. *Suppose* $E|\varepsilon_1|^3 < \infty$, *and for* $1 < p < 2$, $r > 1 + p/2$, *if* $E|\varepsilon_1|^r < \infty$, *then we have*

$$\lim_{\varepsilon \searrow 0} \varepsilon^{2(r-p)/(2-p)-1} \sum_{n=1}^{\infty} n^{r/p-2-1/p} E\{|S_n| - \varepsilon n^{1/p}\}_+ = \frac{p(2-p)}{(r-p)(2r-p-2)} E|Z|^{2(r-p)/(2-p)}, \tag{1.2}$$

where Z *has a normal distribution with mean 0 and variance* $\tau^2 = \sigma^2(\sum_{i=-\infty}^{\infty} a_i)^2$.

Remark 1.1. Let $a_{i+k} = 1, i = k; a_{i+k} = 0, i \neq k, 1 \leqslant k \leqslant n$, then $X_k = \varepsilon_k$, $S_n = \sum_{k=1}^{n} X_k = \sum_{k=1}^{n} \varepsilon_k$. So (1.2) also holds under some suitable conditions when $\{X_k; k \geqslant 1\}$ is a sequence of i.i.d random variables.

Let N be a standard normal variables. Throughout the sequel, C will represent a positive constant although its value may change from one appearance to the next and let $[x]$ indicate the maximum integer not larger than x.

2. Some lemmas

The following lemma comes from Burton and Dehling (1990).

Lemma 2.1. *Let* $\sum_{i=-\infty}^{\infty} a_i$ *be an absolutely convergent series of real numbers with* $a = \sum_{i=-\infty}^{\infty} a_i$ *and* $k \geqslant 1$. *Then*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=-\infty}^{\infty} \left|\sum_{j=i+1}^{i+n} a_j\right|^k = a^k.$$

The next lemma is CLT of moving-average processes.

Lemma 2.2. *Suppose* $\{\varepsilon_i; -\infty < i < \infty\}$ *is a sequence of random variables with* $E\varepsilon_1 = 0$ *and* $E\varepsilon_1^2 = \sigma^2$. $\{X_k; k \geqslant 1\}$ *is defined as* (1.1), *where* $\{a_i; -\infty < i < \infty\}$ *is a sequence of real numbers with* $\sum_{i=-\infty}^{\infty} |a_i| < \infty$. *Then the*