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Asymptotic tail behavior of Poisson shot-noise processes with interdependence between shock and arrival time[☆]

Xiaohu Li[a,b,*], Jintang Wu[b]

[a] Department of Mathematics, University of New Orleans, New Orleans 70148, USA

[b] School of Mathematical Sciences, Xiamen University, Xiamen 361005, China

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ABSTRACT

This paper studies the tail behavior of the Poisson shot-noise processes with interdependent and heavy-tailed random shocks. In the presence of statistical dependence between the shock and its arrival time we establish the asymptotic behavior of the tail probability. Two examples are presented as illustrations of the main results as well.

1. Introduction

In recent decades, the shot-noise process became very popular in various areas, such as insurance risk, credit risk, ad hoc network, queueing and reliability. For a sequence of random variables $\{X_k, k \geq 1\}$, an arrival process $\{\tau_k, k \geq 1\}$, and a real function $g : \mathbb{R}^3 \mapsto \mathbb{R}$, the stochastic process

$$S(t) = \sum_{k\geq 1} g(X_k, t, \tau_k)\mathrm{I}(\tau_k \leq t), \quad t \geq 0, \tag{1.1}$$

includes a number of special shot-noise processes in the literature. For example, $g(x, t, s) = xh(t, s)$ for some $h : \mathbb{R}^2 \mapsto \mathbb{R}$ in Rice (1944) and Schmidt and Stute (2007).

As one of the most useful special case of (1.1), the Poisson shot-noise (PSN) process

$$S(t) = \sum_{k\geq 1} h(X_k, t - \tau_k)\mathrm{I}(\tau_k \leq t), \quad t \geq 0, \tag{1.2}$$

where $\{X_k, k \geq 1\}$ is a sequence of i.i.d random elements in a general measurable space, h is a measurable function such that $h(x, t) = 0$ for any negative time point t, and $\{\tau_k, k \geq 1\}$ is the sequence of arrival times of a homogeneous Poisson process with rate λ. This formulation has been studied in a lot of literature and widely applied in a variety of areas. For example, Lemoine and Wenocur (1986) employed this PSN process to model residual system stress, Brémaud (2000) showed that

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* Corresponding author at: Department of Mathematics, University of New Orleans, New Orleans 70148, USA.
E-mail addresses: mathxhli@hotmail.com, xhli@xmu.edu.cn (X. Li).

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the Lundberg exponential upper bound in the ruin problem of non-life insurance with compound Poisson claims was also valid for this homogeneous PSN processes, and Venkataraman et al. (2006) applied this PSN process to model the network self-interference in an ad hoc network. For more on PSN process and its applications, we refer the readers to Klüppelberg and Mikosch (1995).

Note that the i.i.d assumption for $\{X_k, k \geq 1\}$ is apparently unrealistic in some practical situations. For example, in the insurance of catastrophic risk such as earthquake, hurricane, floods and wild fire, the claim severities X_k's tend to be strongly interdependent, and their tail distributions may not be identical but tail equivalent instead. By the way, it is also restrictive to assume that the sequence of arrival times $\{\tau_k, k \geq 1\}$ are from a homogeneous Poisson process. Recently, in the context of the asymptotically independent random shocks, Weng et al. (2013) studied the tail behavior of the following PSN process by establishing some uniform asymptotic relations for the tail probability,

$$S(t) = \sum_{k\geq 1} X_k h(t, \tau_k) \mathrm{I}(\tau_k \leq t), \quad t \geq 0, \tag{1.3}$$

where $\{X_k, k \geq 1\}$ is a sequence of identically distributed, but not necessarily independent, nonnegative random variables, the *shot* function $h(t, s)$, nonnegative and Borel measurable, represents the cumulative impact on the system up to time t due to each shock, and $\{\tau_k, k \geq 1\}$, independent of $\{X_k, k \geq 1\}$, is another sequence of nonnegative random variables such that $N(t) = \sup\{n : \tau_n \leq t\}$ is a nonhomogeneous Poisson process with intensity $\lambda(t) > 0$ for $t \geq 0$.

It is worth noting that the independence between the shock X_i and its arrival time τ_i is sometimes rather restrictive in practice. Again take the insurance of the natural disasters for example, an earthquake often increases the chance of a by-claim in near future. According to seismology, the next earthquake is likely to be more severe if the previous one occurred a long time ago. In the literature, a lot of studies were concerned about the dependence between the claims and their inter-arrival times. For instance, in Albrecher and Boxma (2004) the time between two consecutive claims depends on the previous claim amount, and in Albrecher and Teugels (2006), the inter-arrival time and the subsequent claim amount are assumed to be interdependent through a copula. Further scenarios of dependence between claim amounts and inter-arrival times were discussed in Biard et al. (2011), readers may refer to Asmussen and Biard (2011) and the references therein for more details in the literature.

To the best of our knowledge, few research works paid attention to the interdependence between a claim and its arrival time in the literature. However, it is not uncommon to come across such situations in insurance practice. For example, according to the historical data, hurricanes in North America was becoming more and more intense in the past several decades. Since the global warming is getting intensified, the succeeding hurricanes and typhoons is incurring more serious damage. As a result, it is reasonable to introduce the positive dependence between the claim amount due to a hurricane damage and its arrival time. Along this line, in this note we have a further study on the PSN in (1.3) in the following two situations: (i) Shocks $\{X_k, k \geq 1\}$ are tail equivalent, upper tail dependent and independent of the arrival process $\{\tau_k, k \geq 1\}$, and (ii) shocks $\{X_k, k \geq 1\}$ are tail equivalent, asymptotically independent and interdependent of the arrival times $\{\tau_k, k \geq 1\}$.

The rest of this paper is organized as follows. For ease of reference we present some preliminaries in Section 2. Section 3 presents the technical assumptions of our model and some lemmas to be used in studying the PSN model. The main theoretical results are developed in Section 4, in Section 5 we present two examples as illustrations of the main theories, and Section 6 concludes the entire paper.

2. Preliminaries

A random variable X or its survival function $\bar{F}(x) = 1 - F(x) > 0$ for all $x \in \mathbb{R}$ is said to be heavy-tailed to the right, or simply *heavy-tailed*, if $\mathrm{E}\left[e^{\beta X}\right] = \infty$ for all $\beta > 0$. In this paper, we will study those regularly varying distributions, a class of heavy-tailed distributions. Formally, a random variable X with survival function $\bar{F}$ has a *regularly varying* right tail of order $\alpha > 0$ (written as $X \in \mathscr{R}_\alpha$) if

$$\lim_{t\to\infty} \frac{\bar{F}(tx)}{\bar{F}(t)} = x^{-\alpha}, \quad \text{for all } x > 0.$$

The random variables with regularly varying distribution are usually employed to describe losses due to catastrophic risks with heavy-tails. For a comprehensive exposition of heavy-tailed distributions and their applications, we refer readers to Resnick (2007).

Definition 2.1 (*Multivariate Regular Variation*)**.** A n-dimensional random vector $\boldsymbol{X}$ is said to be multivariate regularly varying if there exists a function $b(t) \to \infty$ and a nonzero Radon measure μ such that

$$\lim_{t\to\infty} t \cdot \mathrm{P}\left(\frac{\boldsymbol{X}}{b(t)} \in A\right) = \mu(A), \tag{2.1}$$

for every Borel set $A \subset \mathbb{R}^n \backslash \{\mathbf{0}\}$.