



The M -estimator for functional linear regression model

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ABSTRACT

This paper considers the M -estimator for slope function in functional linear regression models. We approximate the slope function by minimizing the loss function based explicitly on functional principal components analysis, and the loss function can be chosen according to what we are estimating. Under mild assumptions, the convergence rate of the estimator of infinite dimensional slope function is derived. A simulation study is conducted to illustrate the numerical performance of the proposed M -estimator.

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1. Introduction

With the rapid development of modern technology, especially the computer-related technique, data can be recorded densely over time (can be seen in the continuum), such as stock and future prices. Data of such type are termed functional or curve data. There are broad application prospects for functional data in a wide range of areas, which promote the development of functional data analysis (FDA). There are vast literature on FDA, such as Ramsay and Silverman (1997, 2002); Ferraty and Vieu (2006); Horváth and Kokoszka (2012).

As an important part of statistics, regression analysis has been expanded to the functional setting. The functional linear regression model with functional predictors and scalar responses has been introduced by Ramsay and Dalzell (1991). In the functional linear regression model, functional coefficients of functional predictors are to be estimated, which is much different from traditional linear models. But there are also some connections. In fact, in semiparametric and nonparametric regression models, functional coefficients appear and many estimators for them are studied. With functional predictors in consideration, we have to transform them based on basis expansion, then the procedure is similar to multivariate problems. At last we reconstruct functional coefficients. During this procedure, it is crucial to choose basis function sequences and a truncating parameter, see Cai and Hall (2006); Hall and Horowitz (2007); Crambes et al. (2009) for details.

Let Y be a real-valued random variable defined on a probability space $(\Omega, \mathcal{B}, P)$, $\{X(t) : t \in F\}$ be a zero mean, second-order stochastic process defined on $(\Omega, \mathcal{B}, P)$ and $EX^2(t) < \infty$ for all $t \in F$. The sample paths of $\{X(t) : t \in F\}$ are in $L^2(F)$, the set of all square integrable functions on F . To express different kinds of functional data, such as curves, images and arrays, F can be subsets of R , R^p or other spaces.

The scalar response Y is supposed to be linearly related to the functional predictor $X(t)$ through the relationship

$$Y = \alpha + \int_F \beta_0(t)X(t)dt + \epsilon \quad (1)$$

where the intercept α and ϵ are scalars, ϵ is a random error variable. The functional coefficient $\beta_0(t)$ is to be estimated. Model (1) is applicable in a wide range of settings, especially where data are collected only through new developments in technology.

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We can express $\beta_0(t)$, $X(t)$ in terms of orthonormal basis chosen independently of the data, but a drawback is that there is no reason why the first elements are used. Therefore, we consider functional regression problems based on functional principal components analysis (FPCA). FPCA has attracted much attention of many statisticians and many theoretical results and practical applications are reported. These researches include contributions to FPCA (see Silverman, 1996; Boente and Fraiman, 2000; Kneip and Utikal, 2001; Hall and Hosseini-Nasab, 2006; Hall et al., 2006; Jiang and Wang, 2010; Berrendero et al., 2011). While other methods could be used, the FPCA technique is currently the most popular. Hall and Horowitz (2007) considered the least square estimator for the functional linear regression model based on functional principal components and obtained the optimal convergence rate of the slope function. Cai and Hall (2006) showed the problems of slope-function prediction have very different characteristics from estimation. There are many more contributions. Crambes et al. (2009) studied smoothing spline estimator for functional linear models. To avoid the problem of an ill-conditioned covariance operator, Ferraty et al. (2012) proposed to modify the FPCA approach by using presmoothing techniques. Yuan and Cai (2010); Cai and Yuan (2012) obtained the optimal convergence rate of the estimator for slope function in the framework of reproducing kernel Hilbert space.

There is a large collection of the functional data literature on the least square estimator of functional linear models. But how to estimate or what types of estimators of the slope function to obtain, depends on what we are interested in. Therefore, other types of estimators should be paid attention to. Motivated by the Nadaraya–Watson kernel estimator, Chen and Zhang (2009); Gheriballah et al. (2013) proposed nonparametric M -estimator. Chen and Zhang (2009) proposed a nonparametric M -estimator of a regression function for stationary dependent processes to achieve desirable robustness properties. Their kernel-based nonparametric estimator was based on local approximation and the structure of $m(x)$ (in their notation) was not specified. Gheriballah et al. (2013) proved the almost complete convergence of their M -estimator under the stationary ergodic process assumption. In this paper we propose M -estimator for the functional linear regression model (1) without specific loss function. According to different interests, we can acquire different estimators of slope function by choosing different criteria functions, which we will discuss below. With some criteria functions, there is not an explicit expression of the estimators for slope function. Thus to obtain the properties of these estimators, we will go around the criteria function and some assumptions about it are necessary.

Besides the functional linear regression models, semi-functional partial linear models, multiple index functional regression models and the functional projection pursuit regression method are also studied (Aneiros-Pérez and Vieu, 2006; Chen et al., 2011; Ferraty et al., 2013). The extension of M -estimator to these models or methods is possible, considering the only difference is that the square loss function is generalized to a convex function $\rho(\cdot)$. However, with general loss functions, some estimators may not be expressed explicitly, thus it may be more difficult in the technical proof and computation. For example, to derive the asymptotic properties of M -estimator for semi-functional partial linear regression models, conditional moment conditions of functional data on real-valued random variable are needed, which is not easy to deal with.

The remainder of the paper is organized as follows. Section 2 introduces the estimating method for functional linear regression models and its computation. The convergence rate of the proposed estimator is given in Section 3. Simulation results are reported in Section 4. The technical details in proofs are presented in the Appendix.

2. Estimation method and algorithm

In Section 1, F is not specified considering different kinds of functional data. In practice, functional data are often valued in well-known common spaces. Here the functional data are curves, and let $F = [0, 1]$, without loss of generality.

Assume that we observe independent and identically distributed data $(X_1, Y_1), \dots, (X_n, Y_n)$ and it is supposed that the error variable ϵ is independent of X and identically distributed. Note that X_i can only be discretely observed, interpolation rule can be adopted to obtain the curves in the preliminary steps.

2.1. Estimation methods

Define the covariance function and the empirical covariance function respectively as

$$K(s, t) = \text{cov}(X(s), X(t)) = EX(s)X(t),$$

and

$$\hat{K}(s, t) = \frac{1}{n} \sum_{i=1}^n X_i(s)X_i(t) - \bar{X}(s)\bar{X}(t),$$

where $\bar{X}(t) = \sum_{i=1}^n X_i(t)/n$, $0 \leq t \leq 1$.

By the spectral expansions of K and $\hat{K}$, we obtain

$$K(s, t) = \sum_{j=1}^{\infty} \kappa_j \phi_j(s) \phi_j(t), \quad \hat{K}(s, t) = \sum_{j=1}^{\infty} \hat{\kappa}_j \hat{\phi}_j(s) \hat{\phi}_j(t)$$

where

$$\kappa_1 > \kappa_2 > \dots > 0, \quad \hat{\kappa}_1 \geq \hat{\kappa}_2 \geq \dots \geq 0.$$

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