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Statistics & Probability Letters 76 (2006) 1245–1254
STATISTICS & PROBABILITY LETTERS
www.elsevier.com/locate/stapro

Incomplete split-plot designs generated from α-resolvable designs

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Received 20 April 2005; received in revised form 29 November 2005
Available online 20 February 2006

Abstract

We present new constructions for incomplete split-plot designs by utilizing the semi-Kronecker product of two certain types of α-resolvable designs. The statistical characteristics of resultant designs are specified by taking advantage of their general balance property. Furthermore, the efficiency factors concerning whole-plot treatments, subplot treatments and interactions are given.

MSC: 62K10; 62K15

Keywords: α-Resolvable designs; Efficiency factors; General balance property; Incomplete split-plot designs

1. Introduction

Let us consider a two-factor experiment of split-plot type in which two factors A and C occur on v_1 levels $A_1, A_2, \ldots, A_{v_1}$ and v_2 levels $C_1, C_2, \ldots, C_{v_2}$, respectively. And let the levels of the factors A and C be the whole-plot treatments and the subplot treatments, respectively. In a split-plot design (SPD), each of b blocks consists of k_1 whole-plots and each whole-plot consists of k_2 subplots. If all the whole-plot treatments occur on whole-plots within each block and all the subplot treatments occur on subplots within each whole-plot, an SPD is said to be of complete type. If an SPD is not of complete type and treatments of each factor for whole-plots and subplots are binary, then it is called an incomplete split-plot design (ISPD).

We consider the statistical properties of ISPDs under a mixed linear model of observations. In the mixed linear model, a three-step randomization is assumed. A three-step randomization means a randomization of blocks, an independent randomization of whole-plots within blocks and an independent randomization of subplots within whole-plots. Thus, in this model, block effects, whole-plot effects and subplot effects are random, and the treatment effects are fixed. A treatment effect is defined by the sum of the factor effects and their interaction effect, and the treatment combinations A_iC_j are arranged in the lexicographical order for $i = 1, 2, \ldots, v_1$ and $j = 1, 2, \ldots, v_2$.

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0167-7152/$ - see front matter
doi:10.1016/j.spl.2006.01.002

We apply the analysis of multistratum experiments possessing the orthogonal block structure proposed by Nelder (1965a,1965b) (cf. also Bailey, 1981; Houtman and Speed, 1983) to ISPDs generated by our construction. For such ISPDs, we consider four strata, i.e., the general area stratum (0), the inter-block stratum (I), the inter-whole-plot stratum (II) and the intra-subplot stratum (III). In the analysis, so-called stratum information matrices play important roles. It is known due to Mejza (1987) that last three strata include the information concerning treatment contrasts. In this paper, equireplicate and proper ISPDs are considered, and their information matrices are given by

$$\begin{aligned} A_1 &= \frac{1}{k_1k_2}N_1N_1' - \frac{r}{v}J_v, \\ A_2 &= \frac{1}{k_2}N_2N_2' - \frac{1}{k_1k_2}N_1N_1', \\ A_3 &= rI_v - \frac{1}{k_2}N_2N_2', \end{aligned} \tag{1.1}$$

where N_1 and N_2 are incidence matrices with respect to treatments vs. blocks and treatments vs. whole-plots, r denotes the number of treatment replications, and I_v and J_v are the identity matrix and the all-one matrix of dimension $v = v_1v_2$, respectively. The eigenvalues of A_1/r, A_2/r and A_3/r are called stratum efficiency factors of the ISPD with respect to a set of treatment effect contrasts defined by eigenvectors of these matrices.

Mejza and Mejza (1996) showed a construction of ISPDs by using the Kronecker product and gave its stratum efficiency factors. Ozawa et al. (2004) presented, by adopting the semi-Kronecker product, a construction for semi-balanced ISPDs (S-BISPDs) and proved the resultant S-BISPDs have a general balance property. They also gave a table of stratum efficiency factors of the ISPDs and further provided a construction for S-BISPDs by using two resolvable balanced incomplete block designs (resolvable BIBDs). However, the ISPDs constructed in Mejza and Mejza (1996) and Ozawa et al. (2004) do not always exist. Even if the ISPDs exist, the replication number of treatments possibly become too large and thus useless in practice.

In this paper, we will give new equireplicate ISPDs by using the semi-Kronecker product of two types of α-resolvable designs which are not necessarily balanced. Moreover, we will characterize the new ISPDs with respect to the general balance property and obtain stratum efficiency factors of the ISPDs.

2. ISPDs generated by two certain types of α-resolvable designs

In this section, defining the semi-Kronecker product and two types of α-resolvable designs, we will discuss general balance property and stratum efficiency factors of new ISPDs.

Let $D = [D_1, D_2, \ldots, D_t]$ and $G = [G_1, G_2, \ldots, G_t]$ be any matrices consisting of t submatrices. Then the semi-Kronecker product (cf. Khatri and Rao, 1968) of D and G is defined by

$$D\tilde{\otimes}G = [D_1 \otimes G_1, D_2 \otimes G_2, \ldots, D_t \otimes G_t],$$

where $\otimes$ denotes the usual Kronecker product of two matrices.

Let N_A and N_C denote the incidence matrices of designs for the factors A and C, respectively. We focus our attention only on α-resolvable designs, i.e., designs whose incidence matrices are represented as follows:

$$N_A = [N_{A1}, N_{A2}, \ldots, N_{At}] \quad \text{and} \quad N_C = [N_{C1}, N_{C2}, \ldots, N_{Ct}].$$

Here, N_{Aj} and N_{Cj} denote the incidence matrices of sizes $v_1 \times \beta_1$ and $v_2 \times \beta_2$, respectively, for the jth replication ($j = 1, 2, \ldots, t$), where $\beta_1 = \alpha_1v_1/k_1$ and $\beta_2 = \alpha_2v_2/k_2$, α_1 and α_2 are the numbers of each treatment of factor A in N_{Aj} and factor C in N_{Cj}, respectively, and $r = t\alpha_1\alpha_2$.

Now we consider an ISPD generated by the semi-Kronecker product of N_A and N_C. The incidence matrix for the ISPD is given as follows:

$$N_1 = N_A\tilde{\otimes}N_C = [N_{A1} \otimes N_{C1}, N_{A2} \otimes N_{C2}, \ldots, N_{At} \otimes N_{Ct}]. \tag{2.1}$$