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Preservation properties of a homogeneous Poisson process stopped at an independent random time

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ABSTRACT

To study the ageing and stochastic properties of lifetime random variables, several classes of lifetime distributions have been defined in reliability literature. In this context, an interesting problem is then to investigate the closure properties of such classes under different operators. In this paper, we study the preservation properties of classes of increasing mean inactivity time (IMIT), increasing variance inactivity time (IVIT), and some other recently introduced classes, such as NRBU, NRBUE, and NBRUrh, under a Poisson process stopped at a random time. We also analyze the preservation properties of Poisson shock models for the above-mentioned ageing classes.

1. Introduction

Recently, some concepts have been introduced in reversed time to analyze the behavior of a lifetime random variable T under the condition that the failure of T has occurred in $[0, t]$. Two important concepts arising in this context are the concepts of reversed hazard rate and the mean inactivity time (MIT). The reversed hazard rate is a dual concept of the hazard rate and the MIT is a dual concept for the mean residual life function. Assuming that the component with lifetime T has failed sometime before t, i.e., $T \leq t$, the inactivity time of T is defined as $T_t = t - T | T \leq t$. The MIT is the expectation of this conditional random variable, i.e., $E(T_t)$. A detailed analysis of the MIT can be found in Chandra and Roy (2001), Finkelstein (2002), and Nanda et al. (2003). The MIT is also known in the literature as the "mean reversed residual lifeör "mean past time". In the discrete case, the MIT, denoted by D-MIT, has been introduced in Goliforushani and Asadi (2008).

Another interesting measure which has been of interest in recent years, is the variance of inactivity time (VIT). The VIT is a dual of the variance of the residual lifetime. Among researchers that deal with the variance of residual lifetime, we refer to Launer (1986), Gupta (1987, 2006), and Gupta et al. (1987). The concept and properties of VIT, which is also known as variance reversed residual life, can be found in Nanda et al. (2003) and Nair and Sudheesh (2010). Furthermore, by analogy with the definition of discrete variance residual life (see Khorashadizadeh et al., 2010) the VIT in the discrete setting can be defined.

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In connection with the MIT, the associated stochastic orders have been considered in Kayid and Ahmad (2004), Ahmad et al. (2005), Ahmad and Kayid (2005), and Asadi and Berred (in press). In the discrete case, the MIT stochastic order is defined in the same way as the discrete mean residual life stochastic order in Shaked and Shanthikumar (2007).

Recently, based on the comparison between the stationary renewal survival of a new device with the parent survival of a used one, new classes of life distributions have been introduced in Abouammoh et al. (2000). These classes are new renewal better than used parent (NRBU), new renewal better than used parent in expectation (NRBUE), and harmonic new renewal better than used parent in expectation (HNRBUE). Li and Xu (2008) have also considered a new ageing notion called new better than renewal used in the reversed hazard rate order (NBRUrh), based on the reversed hazard rate order between a random life and its equilibrium distribution.

Let $\{N(t) : t \geq 0\}$ be a Poisson process with rate $\lambda > 0$ and T a random time independent of the process. It is an interesting problem to study the reliability classes of T that are preserved by the discrete random variable $N(T)$. The random variable $N(T)$ represents the number of random events in the random interval $[0, T]$. In Grandell (1997) and Block and Savits (1980), it has been proved that the main reliability classes of T are inherited by $N(T)$. Moreover, Cao and Wang (1991) and Klefsjö (1982) showed that the classes of life functions NBUC and HNBUE are also preserved. In this work, we explore the above-mentioned property in the reliability classes increasing mean inactivity time (IMIT), increasing variance inactivity time (IVIT), NRBU, NRBUE, and NBRUrh (see Theorems 3.2 and 3.5).

Another issue of interest for two non-negative random variables T_1 and T_2 independent of the Poisson process refers to whether the stochastic orderings between T_1 and T_2 are preserved by $N(T_1)$ and $N(T_2)$. In Shaked and Shanthikumar (2007), there is a comprehensive list of stochastic orders that satisfy the mentioned property. Shaked and Wong (1995) analyzed the preservation of stochastic orderings by a general counting process. In this paper, we also study the preservation of the stochastic order in terms of MIT and VIT by a homogeneous Poisson process (see Theorems 3.3 and 3.4).

On the other hand, preservation properties of Poisson shock model have been widely studied in the literature (see Esary et al., 1973, Klefsjö, 1981, 1983, and Fagiuoli and Pellerey, 1993). The methodology employed in this paper allows us to extend previous results to the classes of lifetime NRBU, NRBUE, and IVIT.

The paper is organized as follows. In Section 2, we present the concepts of mean inactivity time and variance inactivity time together with their associated ageing classes. Furthermore, we establish implications among discrete IMIT and IVIT classes in Lemma 2.6 and define reliability classes NRBU, NRBUE, NBRUrh, and anti-ageing classes. In addition, we consider totally positive functions (see Definition 2.12) and results related to them (see Theorems 2.13 and 2.14) that are useful in the development of the paper. Besides, we present the stochastic orders related to MIT and VIT. The main results of the paper are given in Section 3. The section begins with some auxiliary results and notation for Poisson processes. Closure properties under a Poisson process stopped at an independent time are shown in Section 3.1 for ageing classes IMIT, IVIT, NRBU, NRBUE, and NBRUrh, as well as the preservation of stochastic orders related to MIT and VIT. Finally, in Section 3.2, we deal with preservation properties under Poisson shock models.

2. Preliminaries

From now on, for a non-negative random variable T, let us denote by F_T and $\bar{F}_T$ its cumulative distribution function and reliability function, respectively. Besides, let us denote by $\mathbb{Z}^+$ the set of non-negative integers.

In the next two definitions, we present the concepts of MIT and VIT for a non-negative random variable (r.v.) and a $\mathbb{Z}^+$-valued r.v.

Definition 2.1.

(a) The MIT of T, denoted by ν_T, is defined as follows:

$$\nu_T(t) = E[t - T|T \leq t] = \frac{\int_0^t F_T(u)du}{F_T(t)}, \quad t \geq 0,$$

provided that $F_T(t) > 0$.

(b) The VIT of T, denoted by σ_T^2, is defined as follows:

$$\sigma_T^2(t) = \mathrm{Var}(t - T|T \leq t) = \frac{2}{F_T(t)} \int_0^t \int_0^x F_T(u)dudx - [\nu_T(t)]^2, \quad t \geq 0,$$

provided that $F_T(t) > 0$.

Definition 2.2. Let X be a $\mathbb{Z}^+$-valued r.v.

(a) The discrete mean inactivity time (D-MIT) of X, denoted by $\bar{\nu}_X$, is defined as follows:

$$\bar{\nu}_X(x) = E[x - X|X < x] = \frac{\sum_{k=0}^{x-1} F_X(k)}{F_X(x-1)}, \quad x = 1, 2, \ldots.$$