



Long time behavior of telegraph processes under convex potentials

Joaquin Fontbona^a, H el ene Gu erin^{b,*}, Florent Malrieu^c

^a Center for Mathematical Modeling, UMI 2807 UChile-CNRS, Universidad de Chile, Casilla 170-3, Correo 3, Santiago, Chile

^b Institut de Recherche Math ematique de Rennes (UMR CNRS 6625), Universit e de Rennes 1, Campus de Beaulieu, F-35042 Rennes Cedex, France

^c Laboratoire de Math ematiques et Physique Th eorique (UMR CNRS 7350), F ed eration Denis Poisson (FR CNRS 2964), Universit e Fran ois-Rabelais, Parc de Grandmont, 37200 Tours, France

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Abstract

We study the long-time behavior of variants of the telegraph process with position-dependent jump-rates, which result in a monotone gradient-like drift towards the origin. We compute their invariant laws and obtain, via probabilistic couplings arguments, some quantitative estimates of the total variation distance to equilibrium. Our techniques extend ideas previously developed for a simplified piecewise deterministic Markov model of bacterial chemotaxis.

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* Corresponding author.

E-mail addresses: fontbona@dim.uchile.cl (J. Fontbona), helene.guerin@univ-rennes1.fr (H. Gu erin), florent.malrieu@univ-tours.fr (F. Malrieu).

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1. Introduction

1.1. The model and main results

Piecewise Deterministic Markov Processes (PDMP) have been extensively studied in the last two decades (see [7,8,16] for general background) and have recently received renewed attention, motivated by their natural application in areas such as biology [23,9], communication networks [10] or reliability of complex systems, to name a few. Understanding the ergodic properties of these models, in particular the rate at which they stabilize towards equilibrium, has in turn increased the interest in the long-time behavior of PDMPs.

In this paper we pursue the study of these questions on PDMP models of bacterial chemotaxis, initiated in [11,12] by means of analytic tools, and deepened in [14,21] on simplified versions that can be seen as variants of Kac’s classic “telegraph process” [17].

We consider the simple PDMP of kinetic type $(Z_t)_{t \geq 0} = ((Y_t, W_t))_{t \geq 0}$ with values in $\mathbb{R} \times \{-1, +1\}$ and infinitesimal generator

$$Lf(y, w) = w \partial_y f(y, w) + (a(y) \mathbb{1}_{\{yw \leq 0\}} + b(y) \mathbb{1}_{\{yw > 0\}}) (f(y, -w) - f(y, w)), \quad (1)$$

where a and b are nonnegative functions in \mathbb{R} . That is, the continuous component Y evolves according to $\frac{dY_t}{dt} = W_t$ and represents the position of a particle on the real line, whereas the discrete component W represents the velocity of the particle and jumps between $+1$ and -1 , with instantaneous state-dependent rate given by $a(y)$ (resp. $b(y)$) if the particle at position y approaches (resp. goes away from) the origin. This process describes, in a naive way, the motion of flagellated bacteria as a sequence of linear “runs”, the directions of which randomly change at rates that depend on the position of the bacterium. The emergence of macroscopical drift is expected when the response mechanism favors longer runs in specific directions (reflecting the propensity to move for instance towards a source of nutriments). We refer the reader to [23] for a scaling limit of the processes introduced in [11,12] that leads to simplified models like (1).

In the particular case where the jump rates are constants such that $b > a > 0$, the convergence to equilibrium of the process (1) has been investigated in a previous work [14], where fully explicit and asymptotically sharp (in the natural diffusive scaling limit of the process) bounds were obtained. In the present work we will consider position dependent jump-rates which throughout will be assumed to satisfy:

Hypothesis 1.1. Function b (resp. a) is measurable, even, non decreasing (resp. non increasing) on $[0, +\infty)$, bounded from below by $\underline{b} > 0$ (resp. $\underline{a} > 0$). Moreover we assume that $b(y) > a(y)$ for all $y \neq 0$.

In the sequel, \bar{b} stands for $\sup_{y>0} b(y) \in [\underline{b}, \infty]$ and $\text{sgn} : \mathbb{R} \rightarrow \{-1, +1\}$ denotes the function

$$\text{sgn}(y) = \mathbb{1}_{\{y \geq 0\}} - \mathbb{1}_{\{y < 0\}}.$$

Let us denote by $\mu_t^{y,w}$ the law of $Z_t = (Y_t, W_t)$ when issued from $Z_0 = (y, w)$. The following is our main result:

Theorem 1.2 (Convergence to Equilibrium). *There exists $\kappa > 0$, $K > 0$, and $\lambda > 0$ such that for any $y, \tilde{y} \in \mathbb{R}$ and $w, \tilde{w} \in \{-1, +1\}$,*

$$\left\| \mu_t^{y,w} - \mu_t^{\tilde{y},\tilde{w}} \right\|_{\text{TV}} \leq K e^{\kappa|y|v|\tilde{y}|} e^{-\lambda t}. \quad (2)$$

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